Unfinished in 1977

Note: This draft is unfinished, preliminary and rough. It contains many errors. I would much appreciate it if you would tell me about the ones that you find.

LIMITS ON NEUTRINO BACKGROUND RADIATION

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Abstract

The standard model for the early universe makes no prediction regarding the total number of low-energy neutrinos in the universe. Knowledge of this quantity would be an important guide in such problems as matter-antimatter separation. In this paper many existing observational and experimental results are used to place upper bounds on it. The best limits (mostly not new) come from cosmology and early universe nucleosynthesis. They indicate that less than ten times as many neutrinos as would be expected on the basis of the simplest guess exist. The Fermi energy of a possible degenerate sea of neutrinos is shown from obervations not relying models of the early universe to lie below about 4 keV. For ν_e the best limit is 1.6 eV, while for $\bar{\nu}_e$ it is 35 eV. (Previous limits based on survival of primary cosmic ray protons indicate that $E_F \lesssim 0.3$ eV, but as discussed here, these limits are probably suspect.)

1. INTRODUCTION.

According to the standard 'hot big bang' model of the early universe, the nearly isotropic 3 K themal microwave 'background' radiation now observed has been travelling towards the Earth without appreciable disturbance since its emission some 4×10^5 yrs. after the beginning of the universe [1]. Photons emitted at earlier times will have been absorbed by the rest of the universe, which was then opaque to them. Neutrinos should, however, have passed without interaction from still longer ago. Observation of neutrinos from this period could provide valuable information on the very early history of the universe, and if detected in very large numbers would cast doubt on the validity of the application of General Relativity to the evolution of the universe.

When they were in themal equilibrium with the rest of the universe, the number densities of neutrinos will have been determined simply by the ambient temperature and by their chemical potentials. The chemical potentials cannot yet be estimated on purely theoretical grounds. I shall begin by assuming them to be zero for all species of neutrinos. I also take all neutrinos to be massless. In this case, the present density of each species should be comparable so that of the photon (microwave) background radiation. The most important processes which held neutrinos in themal equilibrium with the electrons and photons in the early universe should have been [1, 5]

$$\nu_i e \to \nu_i e \quad .$$
$$e^+ e^- \leftrightarrow \nu_i \bar{\nu}_i \quad . \qquad (1.1)$$

For types of neutrinos other than ν_e, both these processes occur only through neutral current terms in the weak interaction Hamiltonian. It should be pointed out that although there is extensive experimental evidence for a $\Delta Q = 0$ weak nucleon current, there has only recently been evidence for a neutral weak electron current (from the observation of the process $\nu_\mu e \to \nu_\mu e$[2]), and there is no evidence concerning neutral currents involving neutrinos other than ν_e and ν_μ. Nevertheless, in the simplest gauge theories for weak and electromagnetic interactions, these currents do occur, with roughly equal strengths. The reaction $\nu_i\bar{\nu}_i \leftrightarrow \gamma\gamma$ is forbidden for 'two component' (fixed helicity) neutrinos in the approximation of local interaction[3]. If this idealization is not made, the cross-section in the non-relativistic limit is[4] (m_ℓ is the mass of the charged lepton associated with the ν_i)

$$\sigma\,(\gamma\gamma \to \bar{\nu}_i\nu_i) \simeq 10^{-20}\,[E_\gamma(\mathrm{GeV})]^6 \left(\frac{E_\gamma^4}{M_\ell^4} + 12\,\frac{E_\gamma^2}{M_\ell^2} + 36\right)\,\mathrm{GeV}^{-2}$$

$$= O\left(\frac{E_\gamma^6}{M_W^8}\right) \quad . \tag{1.2}$$

which is negligibly small. The mean free paths of neutrinos undergoing the interactions (1.1) are calculated[5] to exceed the radius of universe (so that the neutrinos cease to be in effective thermal equilibrium with the electrons) when $T \lesssim 10^{10}$ K, which should have occurred about 0.5 s after the beginning of the universe. The electrons dropped out of thermal equilibrium with the photons only at[1] $T \simeq 5 \times 10^9$ K - after the decoupling of the neutrinos.

The specific entropy of a universe filled uniformly with highly relativistic particles in thermal equilibrium at a temperature T is

$$s \simeq \frac{4\pi^2 k^4}{75(\hbar c)^3}\; T^3\, N_{eff}\,(kT) \tag{1.3}$$

where N_{eff} (kT) is the effective number of particle species in thermal equilibrium at temperature T. Boson spin states contribute ½ and fermion ones 7/16 to

N_{eff} (kT). The specific entropy s of the universe is conserved, so that as particle species 'freeze' out of thermal equilibrium (N_{eff} (kT) decreases) the temperature of the universe must rise to compensate. It is believed that at present only the photon background radiation contributes to s, so that $N_{eff} = 1$. At the time of neutrino decoupling, however, electrons (and positrons) will also have contributed to N_{eff} giving $N_{eff} \simeq \frac{4}{11}$. The average temperature when the neutrinos decoupled will therefore have been about $\left(\frac{4}{11}\right)^{1/3}$ of the temperature at the electron decoupling. The present temperature of the neutrino background radiation should therefore be[1, 5]

$$T_\nu \simeq \left(\frac{4}{11}\right)^{1/3} T_\gamma \simeq 1.9 \text{ K}. \qquad (1.4)$$

The higher temperature of the photon background radiation compared to the neutrino one may be thought of as being due to the heating of the universe by the annihilation of e^+e^- after the decoupling of the neutrinos. The present average energy of a neutrino in the thermal neutrino background radiation should be about 2×10^{-5} eV, and the number density of each species of neutrino should be around 1.5×10^{8} m^{-3}. The radiation will evidently contain equal numbers of neutrinos and antineutrinos.

While in thermal equilibrium, the sum of the chemical potentials (μ) of the particles and antiparticles of a particular species must locally be zero. If, however, the numbers of particles and antiparticles in some region differ, then $|\mu| \neq 0$. Such a difference can be maintained if the particles carry a conserved scalar quantum number (or in general only if they are not charge-conjugation eigenstates). The particles and antiparticles of the species will then have opposite values of the quantum number; if the total quantum number for the system is non-zero, then $|\mu| \neq 0$. If the complete system is not in thermal equilibrium, but contains separated regions which are in internal equilibrium, then these too can have $|\mu| \neq 0$. Nucleons in the vicinity of the Earth evidently have $|\mu| \neq 0$. It is possible that neutrinos may exhibit the same phenomenon. If for neutrinos, however, $|\mu| = 0$ locally at all times, then their energy density should be similar to that of the photon background radiation.

Let us assume that the total baryon and lepton (e, μ, τ,...) numbers of the universe are zero. The predominance of matter over antimatter in some regions would therefore presumably be due to its spatial separation in the early universe[6]. Just before neutrino decoupling it would be $\sim 10^4$ times more probable that a neutrino should interact with an electron as with a proton. The separation of protons alone does not therefore provide a simple reason for neutrino separation. Since, however, the neutrino decoupling temperature is not much higher than the e^+, e^- one, it seems possible that the e^+, e^- will already have begun to separate into the $p, \bar{p}$ rich regions by that time. The cross-section for νe scattering is given by[7]

$$\frac{d\sigma\ (\nu e)}{dE_e} \simeq \frac{2\,G_F^2\,m_e}{\pi}\left[A + B\left(1 - \frac{E_e}{E_\nu}\right)^2 - \sqrt{(AB)}\,\frac{m_e E_e}{E_\nu^2}\right]$$

$$\sigma\ (\nu e) \simeq \frac{2\,G_F^2\,m_e\,E_\nu}{\pi}\left[A + B/3 - \sqrt{(AB)}\,\frac{m_e}{2\,E_\nu}\right]$$

$$A\ (\nu_e\, e^-) = \tfrac{1}{4}\,(G_V + G_A + 2)^2 \qquad B\ (\nu_e\, e^-) = \tfrac{1}{4}\,(G_V - G_A)^2$$

$$A\ (\bar{\nu}_e\, e^-) = \tfrac{1}{4}\,(G_V - G_A)^2 \qquad B\ (\bar{\nu}_e\, e^-) = \tfrac{1}{4}\,(G_V + G_A + 2)^2$$

$$A\ (\nu_\mu\, e^-) = \tfrac{1}{4}\,(G_V + G_A)^2 \qquad B\ (\nu_\mu\, e^-) = \tfrac{1}{4}\,(G_V - G_A)^2$$

$$A\ (\bar{\nu}_\mu\, e^-) = \tfrac{1}{4}\,(G_V - G_A)^2 \qquad B\ (\bar{\nu}_\mu\, e^-) = \tfrac{1}{4}\,(G_V + G_A)^2$$

$$A\ (\nu_i\, e^+) = A\ (\bar{\nu}_i\, e^-) \qquad B\ (\nu_i\, e^+) = B\ (\bar{\nu}_i\, e^+)$$

$$A\ (\bar{\nu}_i\, e^+) = A\ (\nu_i\, e^-) \qquad B\ (\bar{\nu}_i\, e^+) = B\ (\nu_i\, e^-) \tag{1.5}$$

where the energies are evaluated in the electron rest frame. For other species of neutrinos, the values of A and B should be the same as those for ν_μ, except if such neutrinos have only V+A couplings, in which case they should be exchanged ($A \leftrightarrow B$). G_V and G_A are respectively the vector and axial vector neutral weak 'charges' of the electron. In the Weinberg-Salam model $G_V = -\frac{1}{2} + 2\sin^2\theta_W$, $G_A = -\frac{1}{2}$, so that with the value $\sin^2\theta_W = 0.25$ favoured by recent experiments, $G_V = 0$ and $G_A = -\frac{1}{2}$. For small E_ν, it is evident that $\sigma(\bar{\nu}e) = \sigma(\nu e)$, but for larger E_ν (as should be important in the case under consideration),

$$\sigma(\nu_i e^-) \simeq \sigma(\bar{\nu}_i e^+) \simeq \lambda_i \, \sigma(\nu_i e^+) \simeq \lambda_i \, \sigma(\bar{\nu}_i e^-)$$

$$\lambda_e \simeq 3 \quad \text{(no neutral currents)}$$

$$\simeq 2.3 \quad \text{(Weinberg-Salam model with } \sin^2\theta_W = 0.25\text{)}$$

$$\lambda_{i \neq e} \simeq \left[\frac{1 + \dfrac{(G_V - G_A)^2}{(G_V + G_A)^2}}{1 + \dfrac{3(G_V - G_A)^2}{(G_V + G_A)^2}} \right] \simeq 1 \qquad (1.6)$$

where the last equality follows if $G_A = 0$ or $G_V = G_A$ or $G_V = 0$, as happens in the Weinberg-Salam model with $\sin^2\theta_W = 0.25$. The preferance of (anti) neutrinos to interact with (anti) matter could cause them to congregate in regions rich in (anti) matter. A quantitative estimate of this effect is difficult.

If neutrinos in some region have a chemical potential $|\mu| >> kT$, then they should form a degenerate 'Fermi sea'. Depending on the sign of μ, this should contain either nearly no neutrinos or no antineutrinos. The other state should fill all the available momentum states up to a momentum p_F ($\simeq |\mu|/c$). Their average momentum would be $p_F/3$, and their number density (assuming them to populate only one helicity state)

$$n \simeq 2 \times 10^{18}\, [p_F\, (eV)]^3\, m^{-3} \quad . \tag{1.7}$$

While there are no theoretical arguments to suggest that a 'big bang' universe should contain degenerate seas of neutrinos, observational evidence can place only somewhat course bounds on n and p_F.

After neutrinos ceased to be in thermal equilibrium with the rest of the universe (or with each other), their numbers could have been increased (or decreased) by non-equilibrium processes. The most important of these is probably neutrino emission from stars. As in the case of photons, however, although particles produced by stars may well have much higher energies than those of cosmological orgin, they should exist only in extremely small numbers. The energy density of neutrinos from stars should be [8] only about 10^{-2} that of cosmological neutrinos even if $|\mu| = 0$.

One model in which most neutrinos would be of non-cosmological origin is an oscillating universe [9], which successively contracts and expands, reaching temperatures only below those at which neutrinos should be in thermal equilibrium. All neutrinos would then come from stellar processes over the complete history of the universe. They would never be in thermal equilibrium, and should form a degenerate Fermi sea. Since the energy of a free relativistic particle in an expanding universe scales as $1/R$, the energies of the neutrinos at the minimun radius of an oscillating universe would be about $\frac{R_{present}}{R_{min}} E_F$ where E_F is their present Fermi energy. Any neutrinos which at that time had energies above their threshold energy E_A for absorption (by matter) will have been removed from the Fermi sea. All lower energy states should, in the course of time, become populated. The typical value of E_A for $\overset{(-)}{\nu}_e$ is around 5 MeV, while for other neutrinos $E_A \sim m_\ell c^2$, where m_ℓ is the mass of the charged lepton associated with them. (Since the ℓ will tend to be unstable, these neutrinos will never be permanently absorbed. Because the universe will have expanded between their absorption and remission, they will, however, tend to is therefore given, in an oscillating cosmology, by [9]

$$E_F = \frac{R_{min}}{R_{present}} E_A \equiv \xi E_A \simeq \frac{T_{present}}{T_{min}} E_A \quad . \tag{1.8}$$

The cosmological bound (to be discussed below) $E_F \lesssim 10^{-2}$ eV for all species of neutrinos then implies [9] $\xi \lesssim 10^{-8}$ from ν_e and $\xi \lesssim 10^{-10}$ from ν_μ. For values of ξ less than about 10^{-4}, the compressed universe will, however, have been sufficiently hot for neutrinos to be in thermal equilibrium with matter, so that their number densities will be roughly those predicted by the simple 'hot big bang' model discussed above, rather than those given by eq. (1.8).

For the remainder of this paper, it will tend to be convenient to specify neutrino background radiations by Fermi energies E_F. The number densities are given approximately by equation (1.7). This expression strictly applies only to a degenerate Fermi sea; for $|\mu| = 0$ one must set $E_F \simeq 10 \langle E \rangle$ in this formula to obtain the correct number density.

2. COSMOLOGICAL CONSTRAINTS

The total present energy density of the universe is an important parameter in the 'hot big bang' model. Neutrinos could make a significant contribution to it. Observational contraints on the total density thus provide constraints on the neutrino density. Accourding to the standard Friedmann model for the universe, General Relativity (with zero cosmological constant) predicts that the deceleration parameter, q, should be given in terms of the present density of the universe, ρ , by[1, 10] (dots devote time derivatives)

$$q = -\frac{\ddot{R}R}{\dot{R}^2} \simeq \frac{\rho}{2\rho_c} \ , \qquad (2.1)$$

$$\rho_c \simeq \frac{3}{8\,G}\left(\frac{\dot{R}}{R}\right)^2 \simeq 2 \times 10^{-26}\ \mathrm{kg\ m^{-3}} \ .$$

If $q \geq \frac{1}{2}$, the universe will expand forever, while if $q < \frac{1}{2}$, it will eventually recontract. If it is assumed that galaxies have had the same average luminosity since the universe was half its present age, then red-shift measurements imply[11]

$$q \simeq 1.0 \pm 0.5 \ . \qquad (2.2)$$

This suggests that

$$2 \times 10^{-26} \lesssim \rho \lesssim 6 \times 10^{-26}\ \mathrm{kg\ m^{-3}} \ . \qquad (2.3)$$

On perhaps a tenth as much matter as would be required to make up this density has ever been observed[1]. The time, t, elaspsed since the beginning of the universe is given by

$$t = \frac{1}{1+\sqrt{q}}\left(\frac{R}{\dot{R}}\right) \qquad (2.4)$$

Red-shift measurement provide a value for the Hubble time[11]

$$\frac{R}{\dot{R}} \simeq (1.3 \pm 0.3) \times 10^{10} \quad \mathrm{yr} \;, \qquad (2.5)$$

while nucleochronology indicates that [12]

$$t \gtrsim 5 \times 10^{9} \quad \mathrm{yr} \;. \qquad (2.6)$$

Combining these with eqn. (2.4), one finds that $q \lesssim 2.6$, indicating that [5]

$$\rho \lesssim 5 \times 10^{-26} \quad \mathrm{kg\ m^{-3}} \;. \qquad (2.7)$$

The density due to a species of massless degenerate neutrinos is given from eqn. (1.5) by

$$\rho \lesssim 10^{-18} \left[E_F\,(\mathrm{eV})\right]^4 \quad \mathrm{kg\ m^{-3}} \;. \qquad (2.8)$$

A single species of massless neutrino must therefore have

$$E_F \lesssim 2 \times 10^{-2} \quad \mathrm{eV} \;, \qquad (2.9)$$

according to the usual cosmological model. In view of the measured value of the deceleration parameter (2.3), observation of a Fermi energy in excess of the bound (2.9) would (if its value remained constant throughout the universe) represent a serious difficulty for Friedmann models of the universe.

A species of neutrino with $|\mu| = 0$ (and $m = 0$) should have

$$\rho \simeq \frac{7}{16}\frac{\sigma}{c^2}\, T_\nu^4 \simeq 5 \times 10^{-32} \quad \mathrm{kg\ m^{-3}} \;, \qquad (2.10)$$

so that up to $\sim 10^6$ such species of neutrino could exist before (2.8) is violated. Another limit on the number of types of massless neutrinos comes from demanding that $\Gamma\,(\psi \to \nu\bar{\nu}) \lesssim \Gamma_{TOT}(\psi)$. In the Weinberg-Salam model

$$\frac{\Gamma(\psi \to \nu_i \bar{\nu}_i)}{\Gamma(\psi \to e^+ e^-)} \simeq \frac{18\, G_F^2 (\sin^2\theta_W - 3/8)^2\, m_\psi^4}{64\, \pi\, \alpha^2}$$

$$\simeq 2 \times 10^{-5} (\sin^2\theta_W - 3/8)^2 \qquad (2.10)$$

$$\simeq 3 \times 10^{-7} ,$$

indicating that the total number of massless neutrinos (which couple to the ψ) is less that about 3×10^7. The absence of a significant $\nu\bar{\nu}$ decay for the Υ (9.5) would set a better limit by a factor of about 100.

If stable massive neutrinos exist, then, so long as they have masses less than about 5 MeV/c^2 [13], they should now be present at their (scaled) equilibrium number density of about 1.5×10^8 $(2s + 1)$ m^{-3} (s is their spin). Each species should contribute a density

$$\rho \simeq 4 \times 10^{-28} (2\,s + 1)\ m\,(\mathrm{eV}/c^2)\ \ \mathrm{kg\ m^{-3}} \qquad (2.11)$$

so that the bound (2.9) suggests [14]

$$\Sigma\, m_\nu \lesssim 100\ \ \mathrm{eV}/c^2\ . \qquad (2.12)$$

Limits on the numbers of degenerate massive stable neutrinos may be obtained by replacing the term $[E_F\,(\mathrm{eV})]^4$ which appears in ρ by $[p_F\,(\mathrm{eV}/c)]^3\, m_\nu\,(\mathrm{eV}/c^2)$.

Big bang nucleosynthesis provides further, more stringent constraints on the neutrino content of the universe [1]. Most of the neutrons in the early universe will either have decayed or been included in ^{4}He. A small number remained in ^{2}H, the number depending drastically on ρ. If it is assumed that the observed interstellar ^{2}H originated in the early univserse, then its abundance perhaps suggests that [15]

$$\rho \quad \simeq \quad 0.03 \; \rho_c \quad \simeq \quad 5 \times 10^{-28} \quad \mathrm{kg\ m^{-3}} \tag{2.14}$$

in superficial disagreement with the value (2.3) obtained from the observed deceleration parameter using a Friedmann model. If some of the ^2H produced in the early universe has been destroyed in stars, then ρ could be much larger than (2.14).

The total number of neutrons which survived decay until the universe became cold enough for them to form ^4He depends on the Fermi energy of a possible $\overset{(-)}{\nu}_e$ Fermi sea [1] . In the period of nucleosynthesis

$$\mu_n \; - \; \mu_p \quad \simeq \quad -\mu_{\nu_e} \quad = \quad \mu_{\bar{\nu}_e} \tag{2.15}$$

since $\mu_e \simeq 0$. The equilibrium neutron fraction is then

$$\frac{n_n}{n_n + n_p} \quad \simeq \quad \left[1 \; + \; \exp\left(\frac{\mu_{\nu_e} \; + \; (m_n - m_p)\, c^2}{k\, T} \right) \right]^{-1} . \tag{2.16}$$

The present ^4He abundance (by mass) is $\simeq 2\, n_n/(n_n + n_p)$. The fact that the observed ^4He abundance does not deviate far from the prediction of (2.16) with $\mu_{\nu_e} = 0$ indicates that at that time

$$E_F^{\nu_e} \quad \sim \quad |\mu_{\nu_e}| \quad \lesssim \quad (m_n - m_p)\, c^2 \quad \simeq \quad 1 \;\; \mathrm{MeV} \tag{2.17}$$

so that the present chemical potential

$$|\mu_{\nu_e}| \qquad \frac{T_\gamma^{now}}{T_\nu^{decoupling}} \; 1 \; \mathrm{MeV} \simeq 2 \times 10^{-5} \; \mathrm{eV} \tag{2.18}$$

Note that, since this value of μ is not far above the average neutrino energy for $|\mu| = 0$, one must distinguish between μ and E_F. In the case of complete degeneracy, of course, $|\mu| = E_F$. More exact calculations [16] show that the predicted

^{4}He abundance remains unaffected (within 5%) only so long as

$$-4 \times 10^{-6} \lesssim \mu_{\nu_e} \lesssim 5 \times 10^{-6} \text{ eV} \qquad (2.19)$$

(negative values of μ represent $\bar{\nu}_e$ degeneracy). If a 10% discrepancy is allowed, both limits become around a factor of 2 larger. The number density[17] of $\overset{(-)}{\nu}_e$ is therefore less than about m^{-3} – close to the $\mu = 0$ value of 1.5×10^8 m^{-3}.

The number of neutrons which survive decay until the universe is sufficiently cold to allow nucleosynthesis will also depend on the rate of expansion (cooling) of the universe. The present abundance (by mass) of ^{4}He is found to depend on the rate of expansion according to the formula[18]

$$\frac{m\,n\,(^4\text{He})}{m\,n\,(^1\text{H})} \simeq 0.268 + 0.0195 \log_{10}\left(\frac{\rho_N}{\rho_c}\right) + 0.190 \log_{10}(N_{eff}) \qquad (2.20)$$

where ρ_N is the present nucleon density, ρ_c is the critical density (eqn (2.1)) and N_{eff} is the effective number of particle species contributing to the energy of the universe at the time of nucleosynthesis. First let us assume that $\mu_\nu = 0$. Any at least nearly massless species of neutrino which undergoes neutral current weak interactions with electrons will have been in thermal equilibrium at that time, and will therefore have contributed around 7/8 to the value of N_{eff}. (This assumes that only one spin state is populated: if more are, this should be multiplied by their number.) The demand that the cosmological abundance of ^{4}He should not differ from its canonical value (obtained from (2.20) by setting $N_{eff} = 9/2$) by more than 5% then constrains the number of such species to be less than about 7[19]. There should thus remain fewer than about three species of (nearly-massless) neutrinos to be discovered. Species of neutrinos which have

$\mu \neq 0$ will contribute more than their $\mu = 0$ value of 7/8 to N_{eff}. A partially or completely degenerate spin state will contribute to N_{eff} a term [20]

$$N_{eff} \simeq \frac{7}{8} + \frac{15}{4\pi}\phi^2 + \frac{15}{16\pi^4}\phi^4, \quad \phi = \frac{\mu}{kT} \qquad (2.21)$$

The constraint that such contributions should not lead to a value of N_{eff} greater than about 4.5 then implies

$$|\mu_\nu| \lesssim 5 \times 10^{-4} \text{ eV} \qquad (2.22)$$

for any massless species of neutrinos. This corresponds to the limit

$$n_\nu \lesssim \qquad (2.23)$$

on their number density. The previous requirement that these particles should have been in thermal equilibrium at the time of nucleosynthesis is no longer effective, since a degenerate species of particles will exist in sufficient numbers to contribute to ρ or s even if it is not in thermal equilibrium.

The bound (2.24) shows that if conventional models for big-bang nucleosynthesis are correct, then there can be only a rather small number of neutrinos in the universe. The remainder of this paper is concerned with limits on the density of neutrino background radiation which do not rely on models of the early universe. None will come even near to the limit form nucleosynthesis (2.22) or from the total mass of the universe (2.9). Nevertheless, their independence of early universe models allows them to serve as an important check on these theories.

3. CONSTRAINTS FROM DECAY PROCESSES.

The presence of large numbers of low-energy neutrinos would, because of the exclusion principle, modify the phase space available for neutrino emission. In the case of a degenerate Fermi sea, no neutrinos (or antineutrinos, depending on the sign of μ) with energies less then E_F could ever be emitted. The resulting modifications to decay energy spectra closely resemble those arising from a finite neutrino mass. Limits on neutrino masses derived from measurements of the minimum neutrino energy in decays are therefore also constraints on E_F [9]. The best limits on the Fermi enrgies (or masses) of the presently-known species of neutrinos derived in this way are [21,22,23,24]

$$
\begin{array}{llll}
E_F^{\bar{\nu}_e} & \lesssim 35 \ \mathrm{eV} & ({}^3H \rightarrow {}^3He\, e^- \bar{\nu}_e) & \\
E_F^{\nu_e} & \lesssim 4 \ \mathrm{keV} & ({}^{22}Na \rightarrow {}^{21}Ne\, e^+ \nu_e) & \\
E_F^{\overset{(-)}{\nu}_\mu} & \lesssim 0.6 \ \mathrm{MeV} & (\mu \rightarrow e\, \nu_e\, \nu_\mu) & \qquad (3.1) \\
E_F^{\overset{(-)}{\nu}_\tau} & \lesssim 0.5 \ \mathrm{GeV} & (\tau \rightarrow \mu\, \nu_\mu\, \nu_\tau) &
\end{array}
$$

where the process in which the measurement was made is given in parentheses.

In addition to depressing the enrgy spectrum for $A \rightarrow B\ell\nu_\ell$ at large E_ℓ, the neutrino background radiation could give rise to processes like $\nu_\ell A \rightarrow B\ell$, yielding $E_\ell > m_A - m_B$, in apparent violation of energy conservation [9]. The fact that no such anomalous events (which would appear to imply a negative neutrino mass) are observed in experiments [21] on ${}^3H \rightarrow {}^3He\, e^- \nu_e$ indicates that

$$E_F^{\nu_e} \lesssim 35 \ \mathrm{eV} . \qquad (3.2)$$

None of the experiments have, however, made a careful analysis of possible neutrino absorbtion (they were designed to detect a $\bar{\nu}_e$ mass) so that (3.2) should be treated with care.

The rates of β decays with very small Q ($=(m_A-m_B)c^2$) values should be increased by absorption of $\overset{(-)}{\nu}_e$ from a neutrino background radiation. The rate for an allowed β-decay with small Q value is given by [25]

$$\Gamma(A \to B\, e\, \nu_e) \simeq \frac{G_F^2\, Q^7}{2\,\pi^3\, m_e^2}\,(.216)\,|M_{if}|^2 \qquad (3.3)$$

The rate of the A → B transition due to neutrino absorption is given by

$$\begin{aligned} \Gamma(\nu_e\, A \to B\, e) &\simeq \sigma(\nu_e\, A \to B\, e)\; n \\ &\simeq \frac{G_F^2\, 2m_A\, E_\nu}{2\,\pi^3}\,|M_{if}|^2\, n \\ &\simeq \frac{G_F^2\, m_A\, E_F^4}{18\,\pi^3}\,|M_{if}|^2 \ , \end{aligned} \qquad (3.4)$$

where the last expression holds for a degenerate Fermi sea of neutrinos. The ratio of the induced to spontaneous transition rate in a degenerate Fermi sea is therefore approximately

$$R \equiv \frac{\Gamma(\nu_e\, A \to B\, e)}{\Gamma(A \to B\, e\ )} \simeq \frac{0.5\, m_A\, m_e^2\, E_F^4}{Q^7} \qquad (3.5)$$

The smallest known Q value in beta decay is for ^{187}Re; the (allowed) β^- decay of this nuclide has Q ≃ 2.6 keV, and the lifetime for the decay is ∿ 4 X 10^{10} yr [26]. Of β^+ decays, the decay of ^{163}Er has the smallest Q value - 191 keV. This nuclide has a mean life of about 75 min [26]. The ratio of neutrino absorption and spontaneous decay rates for ^{187}Re is

To obtain better limits than (3.9) or (3.10) one must find $\beta^{\pm}$ decays with still smaller Q values. An alternative would be to investigate the β decay of a very energetic nucleus[28]. The spontaneous decay rate would decrease like $1/\gamma = mc^2/E$, while the neutrino absorption rate should remain constant as the energy of the nucleus is increased. The apparent laboratory half-life of the nucleus would therefore not decrease as its velocity was increased: an apparent breakdown of the relativistic time dilation formulae. Neutrons are the only β unstable particles with small Q values which may be derived from high-energy proton beams in particle accelerators. The ratio of neutrino absorption to spontaneous decay for a neutron of energy E is

$$\frac{\Gamma(\nu_e\, n \rightarrow p\, e^-)}{\Gamma(n(E) \rightarrow p\, e^-\, \bar{\nu}_e)} \simeq \frac{G_F^2\, E_F\, E\, n\, \tau_{rest}}{3\,\pi}$$

$$\simeq \frac{G_F^2\, E\, \tau_{rest}\, E_F^4}{18\,\pi^3} \qquad \text{(degenerate neutrinos)} \tag{3.11}$$

Even a 1000 GeV neutron beam would therefore only probe $E_F^{\nu_e} \gtrsim 30$ eV.

A similar method cannot be used in μ decay because of its large spontaneous decay rate.

$$R\,(^{187}\mathrm{Re}) \simeq 0.03 \left[E_F^{\nu_e}\ (\mathrm{eV})\right]^4 . \qquad (3.6)$$

The fact that the value of the ^{187}Re β^- half life calculated from nuclear theory[25] does not differ significantly from its measured value suggests that $R\,(^{187}\mathrm{Re}) \lesssim 1$, which implies that

$$E_F^{\nu_e} \lesssim 2\ \mathrm{eV} . \qquad (3.7)$$

A still stricter limit may, however, be obtained. The Fermi level at any time will be given approximately by

$$E_F = E_F^{\mathrm{present}} \frac{R}{R_{\mathrm{present}}} , \qquad (3.8)$$

so that at earlier times, it will have been higher. The ^{187}Re now found was probably formed when the universe was at most half its present age. At that time, E_F will have been larger, and the rate of neutrino absorption higher. The neutrino absorption rate averaged back to the time when the universe had half its present radius should be a factor of about 14/3 higher than the present rate. That the ^{187}Re half-life inferred from galactic nucleochronology[27] does not differ appreciably from that found in laboratory experiments suggests $R\,(^{187}\mathrm{Re}) \lesssim 3/14$, so that

$$E_F^{\nu_e} \lesssim 1.6\ \mathrm{eV} . \qquad (3.9)$$

For the presence of a $\bar{\nu}_e$ degenerate Fermi sea to effect an appreciable decrease in a β^- decay rate, it must have a Fermi energy comparable to the Q value for that decay.

The limit on $E_F^{\bar{\nu}_e}$ obtained from considering the neutrino absorption contribution to ^{163}Er β^+ decay is

$$E^{\bar{\nu}_e} \lesssim 4\ \mathrm{keV} . \qquad (3.10)$$

4. CONSTRAINTS FROM BEAM DUMP EXPERIMENTS.

In this section I describe the only method for detecting neutrino background radiation in which neutrinos from the background radiation may be studied directly. The basic process involved is elastic (neutral current) scattering of a high-energy proton from a low-energy neutrino. Some of the incident proton energy will be transferred to the neutrino, which may then be detected in a bubble chamber. Several experiments have recently been performed in which a high-energy proton beam (typically 400 GeV) impinges on a thick target[29]. Any neutrinos produced in the target pass through to a bubble chamber, in which some of them may be detected by their secondary interactions. The ratio of the number of neutrinos promoted from the background radiation in such an experiment to the number arising from the production and subsequent semi-leptonic decay of a short-lived particle, C, is given by

$$R \equiv \frac{\langle \sigma(p\,\nu \to \nu\, X) \rangle\, n_\nu}{\langle \sigma(p\,N \to C\,X) \rangle\, B(C \to \nu\, X')\ \ n_{\mathrm{nuclei}}}$$

$$\simeq \frac{10^{-18}\left[E_F\ (\mathrm{eV})\right]^4\ \ E_p\ (\mathrm{GeV})}{\langle \sigma(p\,N \to C\,X) \rangle\, B(C \to \nu\, X')\ \ (\mu\mathrm{b})} \qquad (4.1)$$

The latter form assumes a degenerate Fermi sea of neutrinos and an iron target. (Note that to obtain $n_\nu \sim n_{\mathrm{nuclei}}$ one requires $E_F \sim 3$ keV.) Since the process $p\nu \to \nu X$ can occur before the target, while $pN \to CX$ may take place only within it, the rate for the former process will be enhanced relative to that of the latter by a factor $\sim \ell_{\mathrm{beam}}/\ell_{\mathrm{target}}$, where ℓ_{beam} is the distance that the beam travels in line with the target. The neutrino produced in $p\nu \to \nu X$ will always travel in the forward direction, whereas one from the semi-leptonic decay of a particle nearly at rest will be roughly isotropic. This effect should further increase the relative probability of observing $p\nu \rightarrow \nu X$ events. Note that any massless species of neutrino may participate in the process $p\nu \rightarrow \nu X$. The neutrinos produced will have energies up to $\sim E_p$. So long as there is a charged lepton with $m \lesssim E_p$ associated with it, the final high-energy neutrino should undergo the reaction $\nu N \rightarrow \ell^{\pm} X'$ in the analysing bubble chamber, thereby revealing its identity. Results of present beam dump experiments[29] suggest that

$$E_F \lesssim 10 \text{ keV} \tag{4.2}$$

at least for $\overset{(-)}{\nu}_e$ and $\overset{(-)}{\nu}_\mu$ and probably[30] also for $\overset{(-)}{\nu}_\tau$. This limit on the $\overset{(-)}{\nu}_\mu$ Fermi energy is about a factor of 50 lower than the limit from end-points of $\mu \rightarrow \nu_\mu \nu_e e$ energy spectra[23], while one for $\overset{(-)}{\nu}_\tau$ would be a factor $\sim 10^5$ smaller than from $\tau \rightarrow \nu_\tau \nu_\mu \mu$[24]. Unfortunately, until production of charm and other new flavors is determined more accurately in other types of experiment, beam dump experiments cannot be used to place a still more stringent limit on possible neutrino background radiations.

5. CONSEQUENCES OF A 'NEUTRINO AETHER'

The existence of a neutrino background radiation containing a high-density of low-energy neutrinos could cause a number of curious effects. Let us assume that any such radiation was once in thermal equilibrium, and that its composition has remained unchanged since then, so that it will now contain nearly only ν or only $\bar{\nu}$. (If this assumption is relaxed, most of the phenomena described in this section would not occur.) Since neutrinos interact with other particles through a vector field, which can be uncharged, the interaction energies of neutrinos will satisfy (A = e, N, ...)

$$\begin{aligned} \langle \nu A|H|\nu A\rangle &= \langle \bar{\nu}\bar{A}|H|\bar{\nu}\bar{A}\rangle \\ &= -\langle \bar{\nu}A|H|\nu A\rangle = -\langle \nu\bar{A}|H|\nu\bar{A}\rangle \quad . \end{aligned} \tag{5.1}$$

The effective masses of particles and antiparticles in the presence of many ν (or $\bar{\nu}$) will therefore differ. For spin 0 particles

$$\begin{aligned} |m^{A}_{eff} - m^{\bar{A}}_{eff}| &\simeq 2\sqrt{2}\, G_F\, n\, (\hbar c)^3 \\ &\simeq 10^{-24}\, [E_F(\text{eV})]^3 \text{ eV}. \end{aligned} \tag{5.2}$$

A difference in the effective masses of K^o and $\bar{K}^o$ would be manifest in the $K^o_L - K^o_S$ mass difference. The fact that this mass difference is $\simeq 4 \times 10^{-6}$ eV [31] implies that

$$E_F \lesssim 2 \text{ MeV} \quad . \tag{5.3}$$

An effective mass difference $|m_{K^o} - m_{\bar{K}^o}| \simeq 10^{-8}$ eV would be sufficient to account for the observed CP violation effects[32]. However, an effective mass difference arising from interactions with the neutrino background radiation will vary with the velocity of the particle (as γ^2). That no such variation in the strength of CP violation is observed[33] implies

$$E_F \lesssim 200 \text{ keV} . \qquad (5.4)$$

This limit applies to any massless species of neutrinos. The extra effective mass of a particle arising form its interactions with the neutrino background radiation will be a part of its apparent gravitational, but not inertial, mass. Tests of the Principle of Equivalence are now accurate to about one part in 10^{15}, so that

$$E_F \lesssim 1 \text{ MeV} . \qquad (5.5)$$

All ν (or $\bar{\nu}$) of a particular species presumably have the same helicity. Observations of Doppler shifts in the microwave background radiation[34] suggest that the Earth is moving at a velocity of about 2×10^{-3} c with respect to the center of mass of any neutrino background radiation. The two spin states along the direction of the Earth's motion of a spin ½ particle may therefore have different energies as a result of their (usually) neutral current interactions with the neutrino background radiation. The energy difference is[35]

$$|E(\uparrow) - E(\downarrow)| \simeq 2\sqrt{2}\; G_F^{NC}\; n\; \beta_\oplus/(1 - \beta_\oplus^2)^{\frac{1}{2}}$$

$$\simeq 10^{-27} \left[E_F \text{ (eV)}\right]^3 \text{ eV} . \qquad (5.6)$$

This will cause a torque to be exerted on the aligned electrons in a ferromagnet. Such torques have been searched for (to probe possible $\vec{v}_\oplus . \vec{\sigma}_e$ terms violating Lorentz invariance) by looking for a diurnal effect on a ferromagnet suspended inside a solenoid which cancels its field, thereby controlling background magnetic fields. The experimental limit thus obtained is[36]

$$|E(\uparrow) - E(\downarrow)|_e \lesssim 10^{-15} \text{ eV} , \qquad (5.7)$$

implying that for all massless neutrinos

$$E_F \lesssim 10 \quad \text{keV} \quad . \tag{5.8}$$

Another effect[35] of the energy difference (5.6) is to change the relative phases of the helicity states for a propagating particle, thereby rotating its plane of polarization. The rotation angle per unit distance travelled is typically[35]

$$\begin{aligned} \phi/z \quad &\simeq \quad |E(\uparrow) - E(\downarrow)| \quad (1 - \beta_o^2)^{\frac{1}{2}} \quad t/z \\ &\simeq \quad 2\sqrt{2} \quad G_F^{NC} \quad n \\ &\simeq \quad 10^{-17} \quad E_F \text{ (eV)}^{\ 3} \quad \text{rad m}^{-1} \quad . \end{aligned} \tag{5.9}$$

Extra-terrestrial experiments to measure such an effect appear difficult, because of the presence of (time-varying) magnetic field which would tend to swamp it. Observations of muon spin precession in experiments designed to measure the muon anomalous magnetic moment imply that[37]

$$\begin{aligned} |E(\uparrow) - E(\downarrow)|_\mu &\lesssim 4 \quad X \quad 10^{-14} \quad \text{eV} \quad , \\ E_F \quad &\lesssim \quad 30 \quad \text{keV} \quad . \end{aligned} \tag{5.10}$$

An experimental search for diurnal shifts in nuclear magnetic resonance frequencies gives[37]

$$\begin{aligned} |E(\uparrow) - E(\downarrow)|_N &\lesssim 4 \quad X \quad 10^{-17} \quad \text{eV} \quad , \\ E_F \quad &\lesssim \quad 4 \quad \text{keV} \quad . \end{aligned} \tag{5.11}$$

Because of the presumed smallness of the $\gamma\nu_1$ elastic interaction (c.f. eqn. (1.2)), the rotation of the plane of polarization of photons traversing the neutrino background radiation will undoubtedly be very small indeed.

It is believed that atoms may cease to be parity eigenstates because of parity-violating neutral-current weak nucleus-electron interactions. It might be thought that this effect could also be achieved by interactions of the electrons with neutrinos in the background radiation. The usual parity-violating atomic S- and P-wave mixing term due to neutral current interactions of electrons with the nucleus is of the form[38]

$$A^{\text{nucleus}}_{S-P} \simeq C \int \psi^*_S(\vec{r}) \, \frac{\vec{\sigma}.\vec{p}_e}{m_e} \, \psi_P(\vec{r}) \, \delta^3(\vec{r}) \, d^3r$$

$$\sim \frac{d}{dr} \, | \, \psi^*_S(0) \, \psi_P(0) | \, . \qquad (5.12)$$

The mixing amplitude induced by the neutrino background radiation is, however,

$$A_{S-P} \simeq C' \, n \iint \psi^*_S(\vec{r}) \, \frac{\vec{\sigma}.\vec{p}_e}{m_e} \, \psi_P(\vec{r}) \, \delta^3(\vec{r} - \vec{a}) \, d^3r \, d^3a$$

$$\sim \int \frac{d}{dr} \, \psi^*_S(\vec{r}) \, \psi_P(\vec{r}) \, d^3r \qquad (5.13)$$

$$= 0$$

(the angular integral vanishes) so that it cannot in fact induce such mixing. Interactions with the neutrino background radiation can shift energy levels, but the effect is undetectably small.

Neutrinos of very low energy may undergo coherent scattering from the complete volume of a solid scatterer, so that their total interaction cross-section will be proportional to N^2 rather than N[21]. The volume through which the scattering will be coherent is about $\lambda_\nu^3 \simeq 10^{-18} / \left[E_\nu(\text{eV})\right]^3 \; m^3$. If $|\mu|=0$ and $\langle E_\nu \rangle \sim 10^{-5}$ eV, this will be very large. The momentum transferred to the scatterer in each neutrino interaction will however be $\sim E_\nu/c$. Unfortunately, the microwave background radiation and cosmic rays should both exert larger forces than the neutrino background radiation. (If the density of neutrinos is very high, then their energies will be correspondingly large, and their scattering will mostly be incoherent.) Direct mechanical detection of the neutrino background radiation is therefore not possible; instead one must make use of its spin properties, as done above.

6. CONSTRAINTS FROM COSMIC RAY OBSERVATIONS.

High energy cosmic ray particles should lose enrgy by scattering from the neutrino background radiation. A similar phenomenon should occur from the microwave background radiation[39], and the fact that no cut-off to the primary comsic ray spectrum is apparent up to energies $\sim 10^{21}$ eV suggests that they have travelled through the microwave background radiation for $\tau_\gamma \gtrsim 10^{15}$ s. The neutral-current neutrino-proton interaction cross-section in, for example, the standard Weinberg-Salam model is

$$\sigma_{TOT}(\nu\, p) \simeq \frac{10^{-16}\; E_p\,(\mathrm{eV})\; E_\nu\,(\mathrm{eV})}{\left[E_p\,(\mathrm{eV})\; E_\nu\,(\mathrm{eV}) + 10^{22}\right]^2}\; \mathrm{m}^2 \qquad (6.1)$$

where the energies are evaluated in the neutrino rest frame. The interaction time for a proton propagating through a degenerate Fermi sea of neutrinos is

$$\tau_\nu \simeq 1/\sigma\, n\, c$$

$$\simeq \frac{10^{-11}\left[E_p\,(\mathrm{eV})\; E_F\,(\mathrm{eV}) + 10^{22}\right]^2}{\left[E_p\,(\mathrm{eV})\; E_F\,(\mathrm{eV})\right]^4}\; \mathrm{s} \qquad (6.2)$$

For $E_p = 10^{20}$ eV, the condition $\tau_\nu \gtrsim 10^{15}$ s implies[40]

$$E_F \lesssim 0.3\ \mathrm{eV}\ . \qquad (6.3)$$

This extremely stringent bound on the Fermi energies of all types of neutrino (participating in interactions with protons) should not probably be taken very seriously until a cut-off on the cosmic ray spectrum due to interactions with the microwave background radiation is observed. The calculated minimum value of τ_γ is $\simeq 10^{15}$ s, and occurs for $E_p \simeq 10^{21}$ eV, so that failure to observe a cutoff in the spectrum around this energy would prevent this phenomenon from being used to deduce the mean flight time of cosmic rays, and hence their degradation by interactions with low-energy neutrinos.

7. CONCLUSIONS.

Table 1 summarizes limits on the chemical potentials of a neutrino background radiation. It is clear that the best bounds come from cosmology. These indicate that there is no significant excess of neutrinos above the number expected in the standard big-bang model if $\mu = 0$. This result should be explained by any model of matter-antimatter separation in the early universe. Neutrino background radiation, if sufficiently dense, was found to lead to many bizarre effects. Failure to observe these effects sets limits on the chemical potentials (density) of any neutrino background radiation, which, although they are much less stringent then those from cosmology, do not rely on models for early universe nucleosynthesis or of the large-scale structure of the present universe. Refinement of these bounds by further experiment would serve as an important check on cosmological models, and observation of a neutrino background radiation would have profound consequences for models of the universe.

Type(s) of neutrino	Limit on $\lvert\mu\rvert$ (In case of complete degeneracy $\lvert\mu\rvert = E_F$)	Method
All	2×10^{-2} eV	Deceleration parameter - total mass of universe (2)
$\overset{(-)}{\nu}_e$	5×10^{-6} eV	^{4}He production in early universe (2)
All	5×10^{-4} eV	Expansion rate in early universe - ^{4}He production (2)
$\bar{\nu}_e$	35 eV	$^3H \to {}^3He\ e^- \bar{\nu}_e$ end point (3)
ν_e	4 keV	$^{22}Na \to {}^{22}Ne\ e^+ \nu_e$ end point (3)
$\overset{(-)}{\nu}_\mu$	0.6 MeV	$\mu \to e\ \nu_e \nu_\mu$ end point (3)
$\overset{(-)}{\nu}_\tau$	0.5 GeV	$\tau \to \mu\ \nu_\mu \nu_\tau$ end point (3)
ν_e	1.6 eV	^{187}Re neutrino absorption half-life (3)
$\overset{(-)}{\nu}_e$, $\overset{(-)}{\nu}_\mu$, probably $\overset{(-)}{\nu}_\tau$	10 keV	Beam dump experiments (4)
All*	200 keV	K^0 CP violation (5)
All*	4 keV	Diurnal torques in NMR (5)
All*	0.3 eV	Cosmic ray proton survival (limit suspect) (6)

* Must undergo elastic weak interactions with ordinary matter.

Table 1 : Summary of limits on the chemical potentials of the neutrino background radiation. The sections in which each of the limits are discussed are given in parentheses.

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