ABUNDANCES OF NEW STABLE PARTICLES PRODUCED IN THE EARLY UNIVERSE ☆

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The standard model of the early universe is used to estimate the present abundances of possible absolutely-stable hadrons or charged leptons more massive than the proton. It is found that experimental limits on their present abundances indicate that no such particles exist with masses below about 16 GeV/c^2. Forthcoming experiments could increase this limit to masses up to around 300 GeV/c^2.

The standard model of the early universe has recently been used to place constraints on the masses and lifetimes of possible nearly-stable heavy neutrino-like particles predicted by various gauge models of weak interactions [1]. Several models of this kind imply the existence of absolutely-stable charged and/or strongly-interacting particles more massive than the proton (e.g. [2]). In this note, I show that rather large numbers of such particles would have been produced in the early universe, so that experimental limits on their terrestrial abundances may place stringent bounds on their masses.

Any new stable charged particles with masses below about 4 GeV/c^2 should already have been seen in e^+e^- interactions. The next generation of e^+e^- accelerators (PETRA, PEP) could extend this limit to masses up to 20 GeV/c^2. Attempts to produce pairs of new stable hadrons in 400 GeV proton interactions have probed up to masses ≈ 10 GeV/c^2 [2,3], but the production cross-sections for heavy hadrons near threshold are not known with sufficient accuracy for definite conclusions to be drawn [4].

The number density (n) of any species of stable particles spread uniformly throughout a homogeneous universe should obey the rate equation [1,5]

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{-3(\mathrm{d}R/\mathrm{d}t)}{R} n - \langle\sigma\beta c\rangle(n^2 - n_{\mathrm{eq}}^2)\,, \tag{1}$$

where R is the expansion scale factor for the universe and $\langle\sigma\beta c\rangle$ is the product of the low-energy annihilation cross-section and relative velocity for the particles, averaged over their energy distribution at time t. n_{eq} is their number density in thermal equilibrium. The first term in eq. (1) accounts for the dilution in n due to the expansion of the universe, while the second term arises from the annihilation and production of particles in interactions. Let

$$f = \frac{n}{T^3}, \quad x = \frac{kT}{mc^2},$$

$$f_{\mathrm{eq}} = \frac{n_{\mathrm{eq}}}{T^3} = \frac{(2s+1)}{2\pi^2}\left(\frac{k}{\hbar c}\right)^3 \int_0^\infty \frac{u^2\,\mathrm{d}u}{\exp\sqrt{(u^2+x^{-2})} \pm 1}\,, \tag{2}$$

where T is the equilibrium temperature, and in f_{eq} the upper (lower) sign is for fermions (bosons). Then, ignoring the curvature of the universe, which has no effect at the times we consider, eq. (1) becomes ‡1

$$\frac{\mathrm{d}f}{\mathrm{d}x} = Z\,[f^2(x) - f_{\mathrm{eq}}^2(x)]\,, \tag{3}$$

$$k^3 Z = \left(\frac{45}{8\pi^3 G}\right)^{1/2} \frac{m\langle\sigma\beta\rangle}{\sqrt{N_{\mathrm{eff}}(T)}} (c^{11}h^3)^{1/2}$$

$$\approx 4\times 10^{-29} \frac{\langle\sigma\beta\rangle\,[\mathrm{GeV}^{-2}]\, m\,[\mathrm{GeV}/c^2]}{\sqrt{N_{\mathrm{eff}}(T)}}\ \mathrm{GeV}^3\,\mathrm{m}^3\,.$$

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‡1 $\sigma[\mathrm{cm}^2] \approx 4\times 10^{-28}\,\sigma\,[\mathrm{GeV}^{-2}]$.

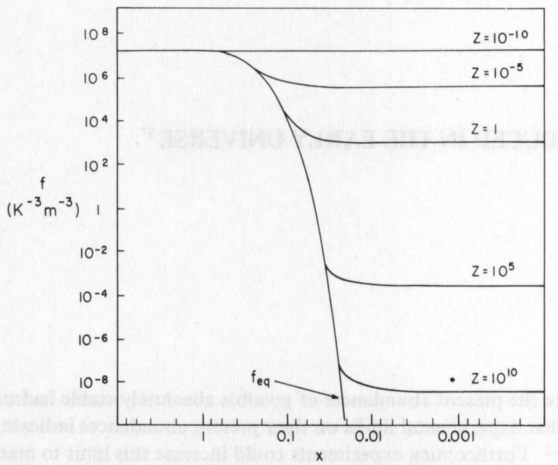

Fig. 1. Solutions to the differential eq. (3) for various values of the parameter Z (in units of $m^3\,K^3$). The curves give the number densities (divided by T^3) of particle species with a variety of low-energy annihilation cross-sections as a function of the average temperature of the universe ($x = kT/mc^2$). The equilibrium number density, $n_{eq} = f_{eq}T^3$, is also given.

If the temperature of the universe was arbitrarily high at early times ‡2 (and the cross-sections for particle interactions do not decrease too rapidly at very high energies), then all particle species should then have been in thermal equilibrium, so that the boundary condition in eq. (1) was $n(t=0) = n_{eq}$ or $f(x=\infty) = f_{eq}$. The solutions of eq. (3) for various values of Z subject to this boundary condition are shown in fig. 1. As the universe cooled, the equilibrium number density of particle species fell dramatically around $x \approx 0.1$. The more strongly-interacting (higher Z) the particles were, the longer they will have remained in thermal equilibrium, and thus the lower their final number density will have been.

The parameter $N_{eff}(T)$ appearing in eq. (3) is the effective number of particle species in thermal equilibrium at temperature T. It determines the energy density and hence the expansion rate of the universe. Ultrarelativistic fermion (boson) spin states contribute 7/16 (1/2) to N_{eff}. (The observed spectrum of particles suggests that for $kT \lesssim 0.1$ GeV, $N_{eff} \approx 4.5$; for $0.1 \lesssim kT \lesssim 0.5$ GeV, $N_{eff} \approx 6$; for $0.5 \lesssim kT \lesssim 2$ GeV, $N_{eff} \approx 35$ (according to QCD quarks and gluons should contribute to N_{eff} as if they were free for $kT \gtrsim 0.5$ GeV); and for $2 \lesssim kT \lesssim 5$ GeV, $N_{eff} \approx 42$.)

The present number density of a particle species is given approximately by $n_p \approx f(0)T_p^3$, where T_p is the temperature which the microwave background radiation would now have if it had frozen out of thermal equilibrium at the same time and temperature ($T_f = mcx_f/k \approx mc^2/[k \log_e(10^{17}(m\langle\sigma\beta\rangle)\,[\mathrm{GeV}^{-1}/c^2\,])]$) as the particle species under consideration. The difference between T_p and the present temperature of the actual microwave background radiation arises from the heating of the universe by the annihilation of other species. Specific entropy conservation gives $T_p \approx T_\gamma/(N_{eff}(T_f))^{1/3}$.

Eq. (3) may be solved approximately by assuming $f = f_{eq}$ for $T > T_f$, and neglecting f_{eq} compared to f for $T < T_f$. This gives

$$n_p \approx \frac{8 \times 10^{-8}}{\sqrt{N_{eff}(T_f)}\,\langle\sigma\beta\rangle\,[\mathrm{GeV}^{-2}]\,m\,[\mathrm{GeV}/c^2]}\ \mathrm{m}^{-3}, \quad (4)$$

which is the correct solution to eq. (3) within about a factor of 20 for the cases considered below.

To obtain estimates of n_p for particular types of particles, one must estimate $\langle\sigma\beta\rangle$. Charged stable heavy leptons ($L^\pm$) with $m_\mu \leqslant m_L \leqslant m_{Z^0}$ should annihilate primarily into two photons, and through a virtual photon to hadrons and lighter leptons, giving

$$\lim_{\beta\to 0}\,(\sigma\beta)_{L^+L^-} \approx \left(\frac{\pi\alpha^2}{m_L^2} + \frac{2\pi\alpha^2}{m_L^2}\,\frac{\sigma_{tot}(e^+e^-)(s=4m_L^2)}{\sigma(e^+e^-\to\mu^+\mu^-)(s=4m_L^2)}\right)\hbar^2. \quad (5)$$

This cross-section, together with the form for N_{eff} discussed above, may now be used to solve eq. (3) and to obtain an estimate for the present abundances of any charged stable heavy leptons. (The exact results are well-approximated by eq. (4).) One finds that for $4 \leqslant m_L \leqslant 10\ \mathrm{GeV}/c^2$, $n_p(L^\pm) \sim 10^{-5}\ \mathrm{m}^{-3}$, corresponding to an abundance of about one new stable charged heavy lepton in 10^5 nucleons. For $m_L \gtrsim 10\ \mathrm{GeV}/c^2$, the estimated present $L^\pm$ number density rises roughly linearly with m_L, except for slight decreases due to increases in the L^+L^- annihilation cross-section associated with the opening of new channels. The abundances of any $L^\pm$ produced in the early universe should therefore be rather

‡2 Models predicting a maximum temperature for hadronic matter are disfavored by recent experimental results indicating the presence of pointlike weakly-interacting constituents within hadrons at short distances.

large, and hence easily amenable to experimental investigation.

To estimate the present abundances of any stable heavy hadrons (H) (containing heavy quarks Q), one must assume a form for the low-energy $H\bar{H}$ annihilation cross-section. An upper bound on $\langle\sigma\beta\rangle_{H\bar{H}}$ is probably provided by the low-energy limit of $\sigma\beta$ for protons [6] $\approx 300\ \mathrm{GeV}^{-2}$. If $m_H \gtrsim 3\ \mathrm{GeV}/c^2$, then the universe at the freezing temperature for the H should have consisted of almost free quarks and gluons, so that a better estimate of $H\bar{H}$ annihilation may be given by the rate for electromagnetic annihilation (5) and for $Q\bar{Q}\to GG$ (obtained from the first term of eq. (5) by replacing α by α_s). The first estimate for $\langle\sigma\beta\rangle_{H\bar{H}}$ leads to $n_p(H) \sim 10^{-11}\ \mathrm{m}^{-3}$ for $m_H = 5\ \mathrm{GeV}/c^2$, decreasing (roughly as $1/m_H$) to $\sim 10^{-12}\ \mathrm{m}^{-3}$ for $m_H = 100\ \mathrm{GeV}/c^2$. The second estimate for $\langle\sigma\beta\rangle_{H\bar{H}}$ suggests $n_p(H) \sim 10^{-8}\ \mathrm{m}^{-3}$ for $m_H = 5\ \mathrm{GeV}/c^2$, increasing roughly as m_H, and perhaps reaching $\sim 10^{-4}\ \mathrm{m}^{-3}$ for $m_H = 100\ \mathrm{GeV}/c^2$. Since it seems most unlikely that the $H\bar{H}$ annihilation cross-section is smaller than its value according to the first estimate, any stable heavy hadrons (with masses below about $100\ \mathrm{GeV}/c^2$) should exist in concentrations above one in about 10^{12} nucleons.

These estimates for heavy hadron abundances may be applied to protons. They give a result $\simeq 10^{10}$ too small. The discrepancy is due to the assumption of homogeneity made in eq. (1); in fact, there must either be a net excess of baryons over antibaryons in the universe, or protons and antiprotons must have become spatially separated (presumably at $kT \gtrsim 50$ MeV) thereby preventing their annihilation [7]. Similar phenomena may have occurred for other stable particles. (An indication that they were not important comes from the result that the present chemical potential (μ) for all species of neutrinos is below 5×10^{-4} eV ‡3, while for $\bar{\nu}_e$, $\mu \leqslant 5\times 10^{-6}$ eV [8].) Inhomogeneity can serve only to increase n_p, so that our estimates should be considered in fact as lower bounds on n_p.

The observed average mass density in the present universe is around $2\times 10^{-26}\ \mathrm{kg\ m}^{-3}$. The requirement that yet unobserved new stable particles produced in the early universe should not contribute a larger mass density than is observed yields (from eq. (4)) $\sqrt{N_{\mathrm{eff}}\langle\sigma\beta\rangle} \gtrsim 7\times 10^{-9}\ \mathrm{GeV}^{-2}$, which is irrelevant for all species of particles except those undergoing only weak interactions [1].

After their production in the early universe, stable heavy particles will presumably have followed the gravitational clumping of ordinary matter. Their number densities should not, however, usually have become sufficiently high for much annihilation to occur. Any L^- produced should have been combined into tightly-bound pL^- systems, while L^+ should occur in $\bar{p}L^+$ or, in the absence of many $\bar{p}$, L^+e^- composites. The fact that the lightest strange and charmed baryons do not undergo strong decay indicates that the lightest baryon carrying a new absolutely-conserved quantum number should not be able to decay into a meson carrying the same quantum number and should therefore be stable. These new stable baryons and mesons should be bound into ordinary nuclei. Any $L^\pm$ and H produced in the early universe should therefore occur in terrestrial material.

Another source of heavy stable particles is pair production by the interaction of cosmic ray particles with the earth's atmosphere. Assuming that all L^+ will eventually get into water, this gives [4] $n_p(L^+) \approx 10^{-22}\ [m_L(\mathrm{GeV}/c^2)]^5$/nucleon ‡4. The cosmic-ray-induced heavy hadron abundance should be about $2\times 10^{-18}\ [m_H[\mathrm{GeV}/c^2]]^{-6}$/nucleon. These abundances are insignificant compared to those expected from heavy particle production in the early universe.

There have been a number of searches for heavy integer-charged stable particles, mostly in sea water. The best published experiment [9] found no such particles in 3×10^{18} nucleons, for almost all masses between 6 and $16\ \mathrm{GeV}/c^2$. When combined with the abundances expected form the early universe, this result suggests that no stable integer-charged particles exist with $1 \lesssim m \lesssim 16\ \mathrm{GeV}/c^2$. The most sensitive search yet made is presently being performed [10] using a mass spectrometer to scan the equivalent of 10^8 kg of sea water. This experiment should detect concentrations down to one new particle in $\sim 10^{20}$ nucleons, for $3 \lesssim m \lesssim 300\ \mathrm{GeV}/c^2$. Modern nuclear physics accelerator techniques, if applied to the same sample, should allow the sensitivity of 10^{-29} new particles per nucleon to be reached

‡3 This result comes from the requirement that the neutrinos should not so alter the expansion rate of the early universe as to affect the amount of ^{4}He produced [8].

‡4 If, however, stable $L^\pm$ can come from the weak decays of hadrons, then their abundances should be comparable to those of their parent hadrons had those hadrons been stable.

[11]. Even if no heavy stable particles were produced in the early universe, a null result in this experiment would show that their abundance was in many cases below that expected just from their production in cosmic ray interactions. The conclusions that no such particles exist (with masses less than several hundred GeV/c^2) would then surely be inescapable, placing an important constraint on present and future models in particle physics.

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