ABUNDANCES OF STABLE PARTICLES PRODUCED IN THE EARLY UNIVERSE

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Abstract

The standard model of the early universe is used to obtain lower bounds on the present abundances of absolutely stable particles. A simple analytic approximation to the results is derived. It is predicted that if there are absolutely stable hadrons or charged leptons more massive than the proton, then they should exist in detectable concentrations, and on this basis experimental results indicate that no such particles exist with masses below $\simeq 16$ GeV/c^2. Forthcoming experiments could increase this limit to masses up to $\simeq 300$ GeV/c^2.

1. Introduction

The purpose of this paper is to present lower bounds on the number densities of stable particles produced in the early universe. I find that there should still exist detectable concentrations of any charged stable leptons with masses of up to perhaps many hundreds of GeV/c^2. The same result probably holds for any stable hadrons.

On the basis of the standard model for the early universe I shall derive a very simple formula that gives approximate lower bounds on the present number densities of any new species of stable particles in the universe which depend essentially on their masses and low-energy annihilation cross-sections. There are as yet no compelling theoretical reasons for the existence of stable particles other than those already known. Numerous further stable particles have, however, been postulated on the basis of various models, and it is obviously of interest to find out if they exist. I shall consider particles to be stable if their lifetimes are comparable with or in excess of the present age of the universe ($\sim 10^{18}$ s). In most schemes, the absolute stability of a particle is a consequence of the fact that it is the lightest particle which carries a particular absolutely-conserved scalar (Abelian) quantum number. It cannot therefore be destroyed in interactions other than annihilation with its antiparticle. I shall consider only particles which behave in this way.

By combining lower bounds on the number densities of possible massive neutrino-like particles with existing cosmological observations, it has already been possible to constrain the masses [1] or lifetimes [2] of such particles. It will turn out that similar constraints do not exist for more strongly interacting stable particles; these are, however, amenable to direct terrestrial experiment.

If and when all the contents of the universe were in thermal equilibrium* the relative number densities of different species of particles were determined simply by the equipartition of energy between them. As the universe cooled and expanded each species in turn froze out of effective thermal equilibrium their mean free paths exceeded the radius of the universe) and their members were destroyed more often than created in interactions. The extent to which the density of the species was diminished by such processes is the most important factor in determining its present density. The more vigorously the particles annihilated, the fewer of them will have survived.

It should be pointed out that the important processes determining the present number density of possible massive stable particles occurred only

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* The existence of a photon background radiation with an apparently thermal spectrum corresponding to a temperature of about 2.7K gives convincing evidence that the temperature of the important contents of the universe was once in excess of about 4000 K. The observed relative abundances of the $A \leqslant 4$ nuclides are consistent with the predictions of the standard model for the early universe indicating that the whole universe was once at the temperature of about 10^9 K necessary for their formation. The detection of a $\overset{(-)}{\nu}_e$ background radiation would serve to reinforce the belief that the universe was once at such a temperature, and observation of a $\overset{(-)}{\nu}_\mu$ background radiation would suggest that it was once hotter than about 10^{12}K. (The simplest estimates indicate that such a neutrino background radiation would have a density comparable to that of the photon background radiation, (rendering it completely undetectable), but it is quite possible (as I shall discuss below) that it is many orders of magnitude denser)

very soon after the beginning of the universe. In fact, a particle species of mass m will have frozen out of thermal equilibrium with the rest of the universe when it was at a temperature of about 5×10^{11} m(GeV/c^2)K (kT $\simeq$ m(GeV/c^2)/20 GeV). This happened about 10^{-4} (m(GeV/c^2) $)^{-2}$ s after the beginning, and when the universe had a mass density of about 10^{16}(m(GeV/c^2)$)^4$ kg m^{-3}. For m $\gtrsim$ 3 GeV/c^2, the universe will at that time therefore have consisted of almost-free quarks and gluons [3] .

Models of the hadron spectrum which imply the existence of a maximum temperature [4] are somewhat disfavoured by recent results on particle interactions at very short distances (large transverse momenta). I shall not consider them here. If, however, the temperature of the universe was, for this or some other reason, never sufficiently high for very massive particle species to be in thermal equilibrium with the rest of the universe, then it is very probable that their present number densities will be negligible*. Nevertheless, if the universe began as a space-time singularity and if present ideas about the behaviour of the strong interactions at short distances are correct, then the universe surely could not fail to have been once at a temperature of at least the Planck temperature (kT $\approx 10^{19}$ GeV).

Particles of cosmological origin might be expected to be spread roughly uniformly throughout the universe. However, with the exception of photons or neutrino-like particles, it seems very likely that they will have become bound to particles of ordinary matter, and will consequently have

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* It is amusing to speculate that one possible such universe might be one which began after the 'bounce' following the contraction of a previous universe. The two universes might follow each other not separated by a space-time singularity. In this case some massive or very weakly-interacting stable particles might survive the hot intermediate stage without interaction. The implications of such models for neutrinos are mentioned in Section 4.

become more concentrated in galaxies and stars. Such clumping undoubtedly occurred long after the number of interactions between the particles was so low as to produce negligible changes in their number density. It is thus reasonable to expect that the relative abundance of nucleons and (charged or strongly interacting) new stable particles will be similar throughout the universe. Temperatures or densities high enough to recreate the conditions in the early universe which caused the number densities of massive particles to change appear not to exist at present over sufficiently large regions to produce a significant effect.

2. The Equation

The number density n of any species of particles spread uniformly throughout a homogeneous universe should obey the rate equation [5,1,2]

$$\frac{dn}{dt} = \frac{-3(dR/dt)}{R}\, n - \langle\sigma v\rangle(n\bar{n} - n_{eq}^2) - \lambda n \qquad (1)$$

where R is the radius of the (matter in the) universe and $\langle\sigma v\rangle$ is the product of the destruction (usually annihilation) cross-section and relative velocity for the particles, averaged over their energy distribution at time t. Since I shall consider only stable particles below, I drop the term $-\lambda n$ which comes from the possibility of their decay. $\bar{n}$ is the number density of the antiparticles of the species under consideration, and n_{eq} is the number density of the particles and antiparticles if they are in thermal equilibrium with the rest of the universe. An equation analogous to equation (1) holds for $\bar{n}$. Until section 4, I shall assume that $n = \bar{n}$. The boundary condition imposed on solutions of equation (1) is that $n = n_{eq}$ for $t \simeq 0$. This corresponds to assuming that at some (perhaps very early) time the particle species under consideration was in thermal equilibrium with the rest of the universe.

The first term on the right hand side of equation (1) takes account of the fact that the volume containing the particles increases as the universe expands. General relativity applied to a universe filled with a uniform gas of highly-relativistic particles gives (e.g. [6])

$$\frac{(dR/dt)}{R} = -\frac{(dT/dt)}{T}$$

$$= \left[\frac{8\pi G\rho}{3c^2} - \frac{\kappa c^2}{R^2}\right]^{\frac{1}{2}} \tag{2a}$$

where κ is the curvature index and G is the gravitational constant. The energy density in the universe, ρ, is given in terms of the average temperature T by (k is Boltzmann's constant)

$$\rho \approx \frac{\pi^2 k^4 T^4}{15(\hbar c)^3} \quad N_{eff}(kT) \tag{2b}$$

where $N_{eff}(kT)$ is the effective number of species in thermal equilibrium at a temperature T. Each fermion or boson spin state contributes $^7/16$ or $^1/2$ respectively to $N_{eff}(kT)$. Near $kT = 0$, N_{eff} should receive contributions from γ, $\overset{(-)}{\nu}_e$, $\overset{(-)}{\nu}_\mu$, $\overset{(-)}{\nu}_\tau$ and $e^\pm$ and be $\simeq 4.5$. At $kT \simeq 0.1$ GeV, it should rise to about 6, as the $\mu^\pm$ begin to contribute. If present beliefs about the behaviour of strong interactions at short distances are correct, then for $kT \gtrsim 0.5$ GeV, quarks and gluons will act as if free [3], and it will be they, rather than the composite hadrons in which they are presently seen to be combined, that contribute to N_{eff}, giving $N_{eff} \simeq 34.5$. The charmed quark and $\tau^{\pm}$ lepton will be in thermal equilibrium for $kT \gtrsim 1.8$ GeV, yielding $N_{eff} \simeq 41.5$. N_{eff} will not rise again until $kT \simeq 4.5$ GeV, at which point it should assume a value around 47. The term in equation (2) which gives the large scale curvature of the universe is not important in the early universe, and will be disregarded below.

The second two terms in equation (1) come respectively from the destruction of particles by annihilation and from their creation by inverse processes. The number density of a particle species with mass m and spin s in thermal equilibrium at a temperature T (and with zero chemical potential) is given by (if M = 0 the factor (2s+1) is replaced by 2s)

$$n_{eq} = \frac{(2s+1)}{(2\pi\hbar)^3} \int_0^\infty \frac{4\pi p^2\, dp}{\exp\left[c\sqrt{m^2c^2+p^2}/kT\right] \pm 1} \qquad (3)$$

where the upper (lower) sign is taken for fermions (bosons). For later convenience I define

$$f_{eq} = \frac{n_{eq}}{T^3} = C \int_0^\infty \frac{u^2\, du}{\exp\left[\sqrt{x^{-2}+u^2}\right] \pm 1} \qquad (4a)$$

where

$$C = \frac{(2s+1)}{2\pi^2} \left(\frac{k}{\hbar c}\right)^3$$

$$\simeq 5 \times 10^6\ (2s+1)\ m^{-3}k^{-3}, \qquad (4b)$$

$$x = \frac{kT}{mc^2}.$$

This is plotted as a function of x in Figure 1. The equilibrium number density of any species of particle is seen to be entirely negligible for $x \lesssim 0.05$*. In Figure 1 I have also plotted the non-relativistic ($mc^2 >> kT$) approximation

$$f_{eq} \simeq \frac{(2s+1)}{\sqrt{2\pi}} \left(\frac{k}{\hbar c}\right)^3 \frac{e^{-1/x}}{\sqrt{x}} \tag{5}$$

which is excellent for $x \lesssim 1$.

In order to simplify equation (1) I shall make the substitutions [5,1]

$$f = \frac{n}{T^3}, \qquad x = \frac{kT}{mc^2}, \tag{6a}$$

in which case it becomes

$$\frac{df}{dx} \simeq \langle\sigma v\rangle \left(\frac{75}{8\pi^3 G}\right)^{\frac{1}{2}} \frac{m}{\sqrt{N_{eff}(x)}} \frac{(c^9\hbar^3)^{\frac{1}{2}}}{k^3} \left[f^2 - f_{eq}^2\right] \equiv Z\left[f^2 - f_{eq}^2\right] \tag{6b}$$

where

$$Z \simeq 6 \times 10^{10} \frac{\langle\sigma\beta\rangle\ (GeV^{-2})\ m(GeV/c^2)}{\sqrt{N_{eff}(x)}}\ m^3K^3 \tag{6c}$$

$$\beta = \frac{v}{c}$$

and f_{eq} is given by equation (4).

The boundary condition for equation (1) now becomes $f = f_{eq}$ for $x \approx \infty$. Note that equation (6) then implies $f \geq f_{eq}$ for all x. This is useful when obtaining numerical solutions. The present number density of

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* The value of x corresponding to a particle species of mass 1 GeV/c^2 at the present is about 2×10^{-13}. The equilibrium number density in the present universe will be negligible except for particles with masses less than about 2×10^{-4} eV/c^2.

a particle species is determined by $f(\frac{kT_\gamma}{mc^2}) \simeq f(0)$ where $T_\gamma \simeq 2.7$ K measures the present temperature of the primeaval fireball. The solutions to equation (6) are evidently functionals of Z, but do not depend separately on any of its component factors. Assuming for now that Z is a constant, one may solve equation (6) numerically. The results are plotted for a wide range of values of Z in Figure 2. (The spin s is taken to be ½, but the results would essentially differ only by the spin multiplicity factor if another value were chosen). The curves in Figure 2 illustrate the important features of solutions to equation (6) which allow one to find simple analytic approximations to them. The rapid fall around $x = 0.1$ signifies the 'freezing' of the particle species out of thermal equilibrium with the rest of the universe. This is apparently a rather rapid process, and the number densities tend to fall from their equilibrium to final values while x (equivalently the average temperature, T) runs over only one or two decades.

3. Approximate Analytic Solutions

In this section I present rather accurate approximate analytic solutions to equation (6) [5,1].

For very light particles, the equation may be solved in a trivial manner. If the present value of x corresponding to the mass of particle under consideration is larger than about 1 then f_{eq} will still be very close to its $x \to \infty$ value of about $(2s+1)10^7\ m^{-3}K^{-3}$. ((2s+1) becomes 2s if the mass is identially zero). For this to be the case one requires $m \lesssim 2 \times 10^{-4}$ eV/c^2.* The value of x at which f drops significantly below f_{eq} will then not yet have been reached, and the particles will still exist at their equilibrium

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* In fact, m may be significantly larger than this before f deviates significantly from f_{eq}. This point is discussed in Section 4.

number density. This situation is relaized in the case of photons, and I have already used this fact in taking $T_\gamma \simeq 2.7$ K to be the present temperature of the primeaval fireball. Presumably the same conditions are satisfied for (light) neutrinos (or other similar fermions) but as will be discussed in Section 4, the equilibrium number densities of equation (3) should in these cases be considered only as lower bounds on the true number densities. Light stable bosons should however behave just like photons for these purposes, and their present number densities should be comparable to that of photons (microwave background radiation).*

Approximate solutions to equation (6) for massive particles may be obtained by solving simplified equations valid in asymptotic regions, and then joining the solutions together. At very large T or x (small times) there will have been genuine thermal equilibrium between the species being considered and the rest of the universe, so that $f = f_{eq}$ then. As T and x decreased, however, there will have come a point at which the equilibrium was destroyed, and for smaller values of x equation (6) may be approximated by

$$\frac{df}{dx} = Zf^2 \qquad (7)$$

since $f \gg f_{eq}$ here. Numerical solutions to equation (6) indicate that the boundary between the equilibrium and non-equilibrium regimes occurs at a 'freezing' value of x, x_f, given approximately by [1]

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* It is amusing to note that since the total decay widths of particles are always less than their masses, sufficiently light particles simply cannot decay (into perhaps photons and/or neutrinos) in the age of the universe. For this phenomenon to be important, however, the particles must have masses less than about 10^{-50} eV/c^2!

$$\left.\frac{df_{eq}}{dx}\right|_{x=x_f} = Zf^2_{eq}(x_f), \tag{8a}$$

which yields*

$$x_F \approx \left[\log_e(10^{17}(m\langle\sigma\beta\rangle)(\mathrm{GeV}^{-1})\,)\right]^{-1} \tag{8b}$$

Then taking $f(x_f) = f_{eq}(x_f)$, equation (7) may be solved to give

$$f(0) \approx \left[x_f Z + \frac{1}{f_{eq}(x_f)}\right]^{-1} \tag{9}$$

To obtain the present number density n from f one must use equation (6a) in reverse to write $n = T^3 f$, where T is some temperature associated with the present universe. Precisely what this temperature is requires discussion. Recall that I took T_γ to be the present temperature of the primeaval fireball. In fact, however, T_γ only represents the average temperature of the universe at the time when photons froze out of thermal equilibrium with matter (plasma recombination time). The correct temperature to use when converting f to n for the present universe is the temperature which the photons would now have if they had dropped out of thermal equilibrium with the rest of the universe at the same time as the particle species under consideration, rather than much later. Their actual temperature differs from that required because of the particle species which froze out of thermal equilibrium between the freezing of the particles being considered and the photons. As each species froze out, it ceased to contribute to the entropy of the universe, and thereby heated it (basically this heating arises from the production of energy by the

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* In all cases of interest, x_f is between about $^1/20$ and $^1/50$.

annihilation of the species). The specific entropy of the universe is

$$s = \frac{4}{3\,T}\,\rho(T) \tag{10}$$

where the energy density ρ is given by equation (2b). It is conserved. As T falls, $N_{eff}(kT)$ will decrease, and to keep s constant T^3 must increase. Recalling that $N_{eff}(kT_\gamma) = 1$ (only photons now contribute to s) the correct equation for the present value of n in terms of f(0) will be approximately

$$n(\text{present}) \approx \frac{T_\gamma^3}{N_{eff}(kT_f)}\, f(0), \tag{11}$$

where T_f is the temperature corre ponding to $x = x_f$.

Combining equations (6c), (9) and (11) one then obtains*

$$n(\text{present}) \simeq \frac{1}{\sqrt{N_{eff}(kT_f)}}\;\frac{3 \times 10^{-8}}{(\langle\sigma\beta\rangle m(\text{GeV}^{-1}/c^2)\ )}\;\; m^{-3}\ , \tag{12a}$$

where the freezing energy kT_f is found from eqn.(8b) to be

$$\begin{aligned} kT_f &= x_f mc^2 \\ &\simeq mc^2/\ \log_e(10^{17}(m\langle\sigma\beta\rangle)(\text{GeV}^{-1}/c^2)) \end{aligned} \tag{12b}$$

Numerical studies indicate that eqn.(12) gives the correct result for the solution of equation (1) (subject, of course, to the assumptions discussed above) within a factor of at most 10. The mass density of a particle species is related to its number density by

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* The $^1/f_{eq}(x_f)$ term in equation (1) can safely be ignored if (as in all calculated cases) $\langle\sigma\beta\rangle \gtrsim 10^{-16}\ \text{GeV}^{-2}$.

$$\rho/c^2 \simeq 2 \times 10^{-27} \; m\,(\mathrm{GeV}/c^2)\; n\,(m^{-3})\;\mathrm{kg\;m^{-3}} \qquad (13)$$

4. Results

The evaluation of the approximate expression (12) or of the differential equation (6) for the present number density of any species of stable particles (subject to the condition $n = \bar{n}$) requires a knowledge of its mass m, spin s and low energy annihilation rate $\langle\sigma\beta\rangle$. (The functional form of $N_{eff}(kT)$ is also in principle required, but its details do not alter the result significantly, and I shall use the guess discussed in Section 2.) The value of $\langle\sigma\beta\rangle$ evidently depends very strongly on the nature of the particles considered. I shall discuss three classes of massive stable particles: charged leptons, neutral leptons and hadrons (by the terms 'hadron' and 'lepton' I mean as usual respectively particles which do and do not participate in strong interactions).

The dominant process by which charged stable leptons, $L^{\pm}$ would be destroyed are depicted in Figure 3. (The diagram of Fig.3(b) with the γ replaced by a Z^{o} will probably become significant only for $m_L >> 100$ GeV/c^2). The low-energy limit of $\sigma\beta$ for the diagrams of Figure 3 is (respectively)

$$\lim_{v\to o}(\sigma\beta)_{L^+L^-} \simeq \frac{\pi\alpha^2}{m_L^{\,2}} + \frac{4\pi\alpha^2}{3m_L^2}\,Q(2m^2_L)$$

$$Q(s) = \frac{\sigma_{TOT}(e^+e^-)(s)}{\sigma(e^+e^-\to\mu^+\mu^-)(s)} \qquad (14)$$

$$(\simeq \theta(s-m^2_e) + \theta(s-m^2_\mu) + R(s)\;).$$

(Note that the cross-section for any exothermic process behaves like $^1/_\beta$ near threshold). The value of Q(s) is known from experiment only for

$s \lesssim 8\ \mathrm{GeV}^2$, but may be guessed up to about $s = 15\ \mathrm{GeV}^2$ (from results on $pp \to \mu^+\mu^- X$). Using this I find that (within a factor of 2) the present number densities of $L^\pm$ should be around $10^{-5}\ \mathrm{m}^{-3}$ for $4 \lesssim m_L \lesssim 10\ \mathrm{GeV/c}^2$ ($L^\pm$ with $m_L \lesssim 4\ \mathrm{GeV/c}^2$ would already have been detected in e^+e^- experiments). This corresponds to an abundance of roughly one $L^\pm$ in 10^7 nucleons. The present average number densities of very massive stable charged leptons depends on the yet unknown spectrum of elementary particles with large masses. Let N_Q be the total number of quark flavours* with masses less than m_L, and let N_L be the number of charged heavy leptons (not counting μ) with such masses. Then the present average number density of $L^\pm$ should be given approximately by

$$n_{L^\pm}(\text{present}) \simeq \frac{1}{\sqrt{N_{eff}(m_L c^2/30)}} \quad \frac{10^{-4}}{11+4N_Q+4N_L}\ m_L(\mathrm{GeV/c}^2)\ \mathrm{m}^{-3} \qquad (15)$$

where a guess for N_{eff} for m_L up to about $200\ \mathrm{GeV/c}^2$ was discussed in Section 2. Equation (15) should overestimate n significantly when $m_L >> m_Z \simeq 80\ \mathrm{GeV/c}^2$. Note that if the $L^\pm$ is the only new fundamental species of particle with a mass larger than about $5\ \mathrm{GeV/c}^2$, then its number density should be larger if m_L is larger. The reason for this curious behaviour is that the annihilation cross-section (14) is inversely proportional to m_L, so that when m_L is larger, fewer $L^\pm$ annihilate before the universe has expanded so much that they rarely meet.

For neutral stable heavy leptons**('heavy neutrinos') $\overset{(-)}{L}{}^o$, the dominant annihilation diagram is believed to be that of Figure 4. This

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* If strong interactions are to be asymptotically free up to indefinitely high energies, then it appears that $N_Q \leq 16$.

** This is the case treated in Refs [1] and [2].

gives*

$$\lim_{v\to o}(\sigma\beta)_{L^o\overline{L}^o} \simeq \frac{16\pi\alpha^2 m_L{}^2}{m_Z{}^4}\,\lambda\,\left[Q(2m_L{}^2)+N_\nu\right] \qquad (16)$$

where λ is a constant, probably of order one, which accounts for the details of the weak interaction, and N_ν is the total number of neutral leptons with masses less than m_L. Assuming (as has been done for N_{eff}) that the only neutral leptons with masses less than about 5 GeV/c^2 are ν_e, ν_μ and ν_τ, and taking $\lambda = 1$, $m_Z = 80$ GeV/c^2, I obtain the values for $n_{\overset{(-)}{L^o}}$(present) given in Figure 5. In the case of neutral stable leptons, the increase of the annihilation cross-section with m_L decreases the number densities of possible very large mass such objects. The analogue of equation (15) is

$$n_{\overset{(-)}{L^o}}(\text{present}) \simeq \frac{1}{\sqrt{N_{eff}(m_L c^2/20)}} \times$$

$$\times \frac{10^3}{2+N_Q+N_L+N_\nu}\,\frac{1}{m_L^3(\text{GeV}/c^2)^3}\ \text{m}^{-3} \qquad (17)$$

The present average mass density of the universe is believed to be between about (e.g. [6]) 10^{-25} and 10^{-28} kg m^{-3}, and the requirement that neutral stable heavy leptons should not contribute a mass density far in

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* I have here assumed that the ambient temperature is lower than is necessary to restore the presumably spontaneously broken symmetry between the weak and electromagnetic interaction. Simple estimates suggest that this temperature is $kT \simeq 300$ GeV.

excess of this constrains [1,3] $m_L \gtrsim 2$ GeV/c^2. For heavy particles other than neutral leptons, such constraints are irrelevant, since they demand

$$n \lesssim \frac{10}{m(\mathrm{GeV}/c^2)}\, m^{-3},$$

or (using equation (12))

$$\langle\sigma\beta\rangle \gtrsim 10^{-7}\ \mathrm{GeV}^{-2},$$

which is inevitably satisfied.

I shall now consider the abundance of protons in the universe. An extrapolation of low-energy data [8] on $\sigma_{TOT}(p\bar{p})$, which extends down to kinetic energies of order 40 MeV, indicates that*

$$\lim_{v\to o}(\sigma\beta)_{p\bar{p}} \approx 300\ \mathrm{GeV}^{-2} \qquad (18)$$

Equation (1) (or the approximation (12)) then yields

$$n_{p,\bar{p}} \simeq 1 - 4 \times 10^{-11}\ m^{-3}, \qquad (19a)$$

Corresponding to a mass density

$$(\rho/c^2)_{p,\bar{p}} \simeq 2 - 8 \times 10^{-38}\ \mathrm{kg\ m}^{-3}. \qquad (19b)$$

Observations indicate however, that

$$(\rho/c^2)_{p,\bar{p}} \simeq 10^{-25} - 10^{-28}\ \mathrm{kg\ m}^{-3}. \qquad (20)$$

There appear to be two possible explanations for this huge discrepancy [9] (in the framework of the model discussed here). First, the universe could have a net (probably positive) baryon number, so that some of the protons

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* The cross-section near, rather than actually at rest is in fact needed, so that the Coulomb effects expected there may be disregarded.

simply did not have antiprotons to annihilate with. This seems a somewhat unnatural hypothesis, in view of the fact that while the protons were still in thermal equilibrium with the rest of the universe, requires $(n_p - n_{\bar{p}})/(n_p + n_{\bar{p}}) \sim 10^{-8}$ (see below). Alternatively the assumption that the universe is homogeneous (which was made in applying equation (1) for the average number density) could be incorrect, at least for protons. Specifically, the protons and antiprotons could at some time have become spatially separated, so that while some parts of the present universe are enriched with protons, others are enriched with antiprotons. The electrons and positrons in the early universe should follow any separation of charge on nucleons, so that the present electron number density will be closely linked with the proton one. If complete separation of p and $\bar{p}$ occurred while the nucleons were still in thermal equilibrium with the rest of the universe, then their present density should be simply their high-temperature equilibrium one corrected for the expansion of the universe:

$$(\rho/c^2)_{p,\bar{p}} \simeq 10^{-18} \text{ kg m}^{-3}. \qquad (21)$$

This value is far too large, suggesting that separation occurred only after the nucleons began to freeze out of thermal equilibrium. Figure 6 shows the predicted number density of nucleons (divided by T^3) as a function of T in a homogeneous universe (with $n(T{=}\infty) = \bar{n}(T{=}\infty)$). On the basis of this the observational result (20) indicates that separation should have occurred at $T \simeq 5 \times 10^{11}$ K ($kT \simeq 50$ MeV).

The fact that one has little idea why this separation took place (if indeed it did) means that one cannot tell whether similar phenomena effected particles other than nucleons. It is clear, however, that any effect which inhibits the annihilation of particles will serve only to increase their present number density. Thus equation (12) should be taken as a lower bound on the number densities of particle species, and it should be remembered that in the only (non-trivial) case amenable to observation, it gives a result too small by a factor of more than 10^{10}. A larger number of leptons than protons should survive annihilation in a homogeneous universe, perhaps suggesting that separation would be less important in the former case. It is nevertheless interesting to note that such a separation could produce some curious effects. As mentioned in Section 3, very light particles should exist at their equilibrium number densities. For fermions (or bosons carrying a conserved quantum number) the equilibrium number density depends not only on their temperature, but also on their chemical potential. If there are equal numbers of particles and their antiparticles, the chemical potential will be zero, but if for some reason the particles and antiparticles become spatially separated (or if there is simply a global excess of one over the other) non-zero local chemical potentials (μ) will develop. The number densities in regions of finite μ can exceed the $\mu = 0$ value. For example, at T=0, the number density of a ('degenerate') system of massless fermions is given by

$$n = \frac{gE_F^3}{6\pi(\hbar c)^3} \simeq 2 \times 10^{18} gE_F^3 \text{ (eV)}^3 \text{ m}^{-3} \qquad (22)$$

$$g = 2s + 1 \quad \text{for} \quad m \neq 0,$$
$$= 2s \quad \text{for} \quad m = 0,$$

where E_F is the energy of the most energetic particle in the system (Fermi energy). (For bosons $n = 0$ at $T = 0$ even if $\mu \neq 0$). The constraint that any such massless degenerate fermions do not give rise to a mass larger than the supposed total mass of the universe implies

$$E_F \lesssim 10^{-2}\ \text{eV}, \qquad n \lesssim 10^{12}\ \text{m}^{-3}. \tag{23}$$

Present limits on the Fermi energies for neutrinos are all far in excess of this bound. (Large numbers of neutrinos can arise in oscillating cosmologies [10] . In such models the neutrinos are never in genuine thermal equilibrium with the rest of the universe: they are produced simply in nuclear processes over the complete history of the universe. In this case [10], $E_F^{\nu} \sim E_A R_{minimum}/R_{present}$ where E_A is the threshold for absorption of the neutrinos by matter, and R is the radius of the universe). If the masses and chemical potentials of all neutrinos vanish, then, in the standard model, the present number density of each species should be its (expansion-corrected) equilibrium number density $n \approx 3 \times 10^8\ s\ \text{m}^{-3}$, and its average energy about 2×10^{-5} eV. Neutrinos with small, but finite, mass should exist at a concentration $n \approx 1.5 \times 10^8\ (2s + 1)\ \text{m}^{-3}$ and should contribute $\rho/c^2 \approx 4 \times 10^{-28}\ (2s + 1)\ m\ (\text{eV}/c^2)\ \text{kg m}^{-3}$ to the density of the universe. The probably miniscule value of $<\sigma\beta>$ even for massive neutrinos causes their number density ($/T^3$) not to deviate significantly from its equilibrium value. As can be seen from Figure 5, only neutrinos with $m \gtrsim 5\ \text{MeV}/c^2$ have sufficient $<\sigma\beta>$ to effect an appreciable diminution. The constraint that the density of low mass neutrinos should not exceed the apparent total density of the universe (20) then implies [11] that $\sum m_{\nu} \lesssim 100\ \text{eV}/c^2$, where the sum is taken over all species of neutrinos with $m \lesssim 5\ \text{MeV}/c^2$.

An estimate of the present abundance of possible stable hadrons, H, more massive than protons [12] requires knowledge of their low energy annihilation cross-section. One (necessarily completely unreliable) guess at this may be made by supposing that the diagram of Figure 4(a) with the photons replaced by gluons accounts for the annihilation cross-section.

This suggests*

$$\lim_{v \to o} (\sigma\beta)_{H\bar{H}} \simeq \frac{\pi \alpha_s^2 (m_H^2)}{m_H^2} + \text{electromagnetic part} \qquad (25)$$

$$\alpha_s(Q^2) \simeq \frac{0.5}{1+0.41\log(Q^2(\mathrm{GeV}^2)/4)} .$$

This formula leads to the prediction that the present H number density should be about 10^{-8} m^{-3} for $m_H \simeq 5$ GeV/c^2, increasing to about 10^{-6} m^{-3} for $m_H \simeq 10$ GeV/c^2 and perhaps reaching 10^{-4} m^{-3} at $m_H \simeq 100$ GeV/c^2. It seems likely that $(\sigma\beta)_{H\bar{H}} < (\sigma\beta)_{p\bar{p}}$, so that by taking $(\sigma\beta)_{H\bar{H}} = (\sigma\beta)_{p\bar{p}}$ one should obtain a lower bound on n_H. This gives $n_H \simeq 10^{-11}$ m^{-3} for $m_H \simeq 5$ GeV/c^2, decreasing to $n_H \simeq 10^{-12}$ m^{-3} for $m_H \simeq 100$ GeV/c^2. Even this rather conservative estimate therefore suggests that if stable heavy hadrons exist, then they should be at a detectable concentration in the present universe. Without a better idea of their low energy annihilation cross-section it is however not possible to make precise predictions for their probable abundance.

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* For new hadrons more massive than about 3 GeV/c^2, present theories of the strong interactions at high energies indicate that annihilation will have occurred before most of the quarks 'condensed' into hadrons, perhaps favouring this guess.

5. Local Abundances

Although, as mentioned in Section 1, there should not be regions of sufficiently high temperature and density in the present universe to create or destroy stable particles in anything like the quantities in the early universe, effects in the present universe will undoubtedly determine for example on which elements possible stable charged heavy particles would concentrate. Other authors [13] have investigated the behaviour of neutral stable heavy leptons in the present universe: I shall not consider this here. Instead I shall concentrate on charged stable heavy leptons, $L^{\pm}$, and on stable heavy hadrons, H. I discuss the latter first.

In the case of both strangeness and charm, the lightest baryons carrying the quantum number do not undergo strong decay. It seems reasonable to suppose, therefore, that the lightest baryon carrying a possible absolutely-conserved new flavour would be entirely stable. When the temperature in the early universe fell below about 5×10^8K ($kT \sim 500$ MeV) present theories of the strong interactions suggest that the gas of almost-free quarks which existed at higher temperatures [3] should 'condense' into hadrons. As mentioned in Sections 3 and 4 this phenomenon will probably have occurred only after the annihilation period for sufficiently massive new hadrons ($m \gtrsim 3$ GeV/c^2). Whether the hadrons formed in the 'condensation' are predominantly mesons or baryons cannot yet be calculated. Nevertheless, even if it is mostly mesons which are produced at this stage, they will undoubtedly undergo strong interactions with the nucleons in the early universe to form the new stable baryons considered above. So long as the charge of the new quark is $\frac{2}{3}e$ or $-\frac{1}{3}e$, the baryons will participate in cosmological nucleosynthesis in much the same way as ordinary nucleons. It is not clear, however, that they would have separated from their antiparticles in the same way. It also seems quite likely that the new baryons may have slightly weaker strong interactions than nucleons, which would inhibit their inclusion in nuclei other than hydrogen. When material containing the new baryons was inside stars, it should, at least to some extent, have undergone nucleosynthetic interactions, thereby creating some heavy nuclei containing new particles. The details of the process would probably be very different from those of the standard one. Thus possible stable heavy hadrons should be found roughly equally in the various elements, but probably more in hydrogen.

Possible stable charged heavy leptons, $L^{\pm}$, could be bound to nuclei only through Coulomb interactions. If the $L^{\pm}$ remained mostly outside the nucleus, then the ground state binding energy would be approximately

$$E_B \simeq 27 \quad \mu(\mathrm{GeV}/c^2)\ Z^2\ \mathrm{keV}$$

$$\mu = \frac{m_L m_{\mathrm{nucleus}}}{m_L + m_{\mathrm{nucleus}}} \qquad (26a)$$

(corresponding to a temperature $T \simeq 3 \times 10^8\ \mu(\mathrm{GeV}/c^2)\ Z^2\ \mathrm{K}$) and the ground state probability density would peak at a radius of about

$$a_L \simeq \frac{25}{\mu(\mathrm{GeV}/c^2)Z} \times 10^{-15}\ \mathrm{m} \qquad (26b)$$

Thus for $Z \gtrsim 5$, the $L^{\pm}$ would spend most of their time inside the nucleus, and their binding energy would be reduced from the value given by eqn(26a). It will also then be roughly independent of Z. Around the time of ^{4}He formation (and very probably after nucleon-antinucleon separation), the $L^{\pm}$ will have become bound to nucleons. Since the probability of binding is proportional to Z^2, and the abundance of ^{4}He about $^1/16$ (by number), about $^3/4$ of the $L^{\pm}$ will initially have formed $L^{\pm}\overset{(-)}{p}$ composites. Effectively no $L^{\pm}e^{\mp}$ should form. However, the $L^{\pm}\overset{(-)}{p}$ binding energy given in equation (26a) is such that these composites will often be broken up by the temperatures typically found in stars. The $L^{\pm}$ will then become redistributed, preferentially on to nuclei with high Z. Since the abundances of nuclides decrease rapidly with A, the redistribution may, however, not be very effective in removing the $L^{\pm}$ from hydrogen. The large value of a_L (eqn.(26b)) means that $L^{\pm}$ should not reduce greatly the Coulomb barrier for nuclear interactions of pL composites, and for systems containing heavier nuclei, the fractional decrease in the barrier will be small. Nevertheless, differential binding of the $L^{\pm}$ to nuclei of different Z will ruin usual α and $\beta^{\pm}$ stabilities. Despite this, it seems probable that reactions inside stars will tend to form at least some nuclei heavier than hydrogen containing $L^{\pm}$.

The conclusion of this section is therefore that any stable heavy charged leptons or hadrons produced in the early universe will probably be found roughly equally on all elements.

5. Conclusions

In this paper I have used the standard model of the early universe (albeit for somewhat earlier times than those for which it is usually applied) to obtain lower bounds on the present number densities of stable particle species. For protons they give a result $\sim 10^{10}$ times smaller than the observed number density. The bound may be applied to possible stable particle species other than those already known. For neutral heavy leptons it reproduces earlier results [1,3]. Its application to stable charged heavy leptons indicates that if such particles exist, they should be present in matter at very high concentrations ($\gtrsim$ 1 in 10^9 nucleons*) if they have masses up to even hundreds of GeV/c^2. Ignorance of the low-energy

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* I have assumed here that the average number density of nucleons in the present universe is ~ 10 m^{-3}.

annihilation cross-section for possible stable heavy hadrons prevents a reliable numerical estimate of their present abundance. Simple guesses suggest, however, that if they exist these too should be present in comparatively large numbers (1 in 10^{13} nucleons).

The interactions of cosmic ray particles with the earth's atmosphere should give rise to insignificant numbers of possible massive stable particles compared to the numbers which should have been produced in the early universe. If one makes the assumption that any positively charged new stable particles from cosmic ray interactions eventually get into water, then this will result in an L^+ abundance of about $10^{-22}\,[m_L(\mathrm{GeV}/c^2)]^{-5}$/nucleon.* The cosmic-ray-induced hadron abundance should be about $2 \times 10^{-18}\,[m_H(\mathrm{GeV}/c^2)]^{-6}$ /nucleon. These results will be derived and discussed in a forthcoming paper [14] .

There have been a number of experimental searches for possible massive stable charged particles (assumed here to have charges which are integer multiples of the electronic charge). The best published limit is due to Alväger and Naumann [15] , who found no such particles in 3×10^{18} nucleons from water. Their search used a mass separator on samples of D_2 and D_2O. These samples were already enriched in possible heavy particles by a factor of 10^3 - 10^4 over the water from which they were distilled. The experiment was sensitive to particles with nearly all masses between 6 and 16 GeV/c^2. When combined with the abundances expected from their production in the early universe, its null result then strongly suggests that no stable

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* If, however, stable $L^\pm$ can come from the weak decays of hadrons, then their abundances should be comparable to those of their parent hadrons had those hadrons been stable.

charged particles exist with $1 \lesssim m \lesssim 16$ GeV/c^2.

If possible new stable charged particles had sufficiently small masses, then they should already have been detected in accelerator experiments. Since the production cross-section for a new stable charged particle in e^+e^- annihilation would undoubtedly be comparable to the $e^+e^- \to \mu^+\mu^-$ cross-section, the failure to observe such particles near threshold in experiments where particle velocities can be measured indicates that they do not exist with masses less than about 4 GeV/c^2. There have been various attempts to produce new stable particles with high-energy hadron beams. They are summarised in Ref. [16]. The production cross-sections to which they are sensitive tend, however, to be far above even those estimated for possible massive (stable) hadrons [17].

The most sensitive search for possible stable charged particles yet made is presently being performed [18] using a mass spectrometer to scan a sample of water enriched in heavy objects by a factor about 10^{11} (before enrichment the sample consisted of about 10^8 kg of water). The experiment should detect concentrations down to one new particle in $\sim 10^{20}$ nucleons [19] of the original water, for masses in the range 3 to 300 GeV/c^2 (excluding the masses of ordinary nuclei present in the sample). Modern nuclear physics accelerator techniques involving particle identification methods, if applied to the same sample, should allow the sensitivity of 10^{-29} new particles per nucleon (in water) to be reached [20]. Even if no massive stable charged particles were produced in the early universe, a null result in this experiment would show that their abundance was in many cases below that expected just from their production in cosmic ray interactions [14]. The conclusion that no such particles exist (with masses less than several hundred GeV/c^2) would then surely be inescapable, placing an important constraint on present and future models in particle physics.

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Figure captions

Figure 1: The equilibrium number density (divided by T^3) for a species of particle with mass m as a function of $x = kT/mc^2$. A nonrelativistic approximation, valid when $kT << mc^2$, is also shown. The number densities for fermions and bosons differ essentially only by their spin multiplicity factors.

Figure 2: Solutions to the differential equation (6) for various values of the parameter Z. The curves give the number densities (divided by T^3) of particle species with a variety of low-energy annihilation cross-sections as a function of the average temperature of the (assumed homogeneous) universe ($x = kT/mc^2$). The equilibrium number density, f_{eq}, is also shown.

Figure 3: The dominant mechanisms for the annihilation of charged heavy lepton pairs.

Figure 4: The dominant process by which neutral heavy lepton pairs should annihilate.

Figure 5: The lower bound on the present number density of possible stable neutral heavy leptons as a function of their mass.

Figure 6: The number density (divided by T^3) of protons as a function of the average temperature in a homogeneous universe.

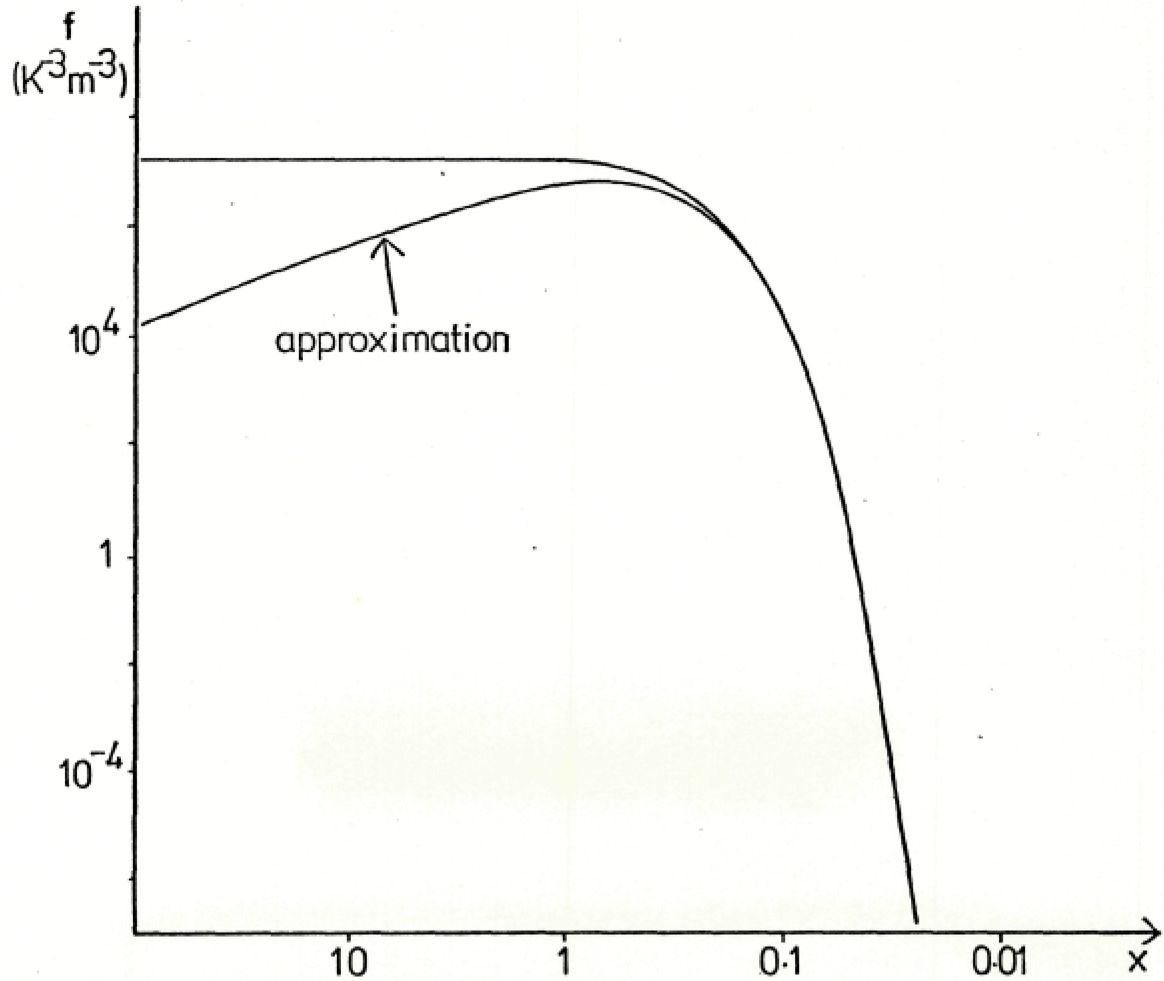


FIGURE 1

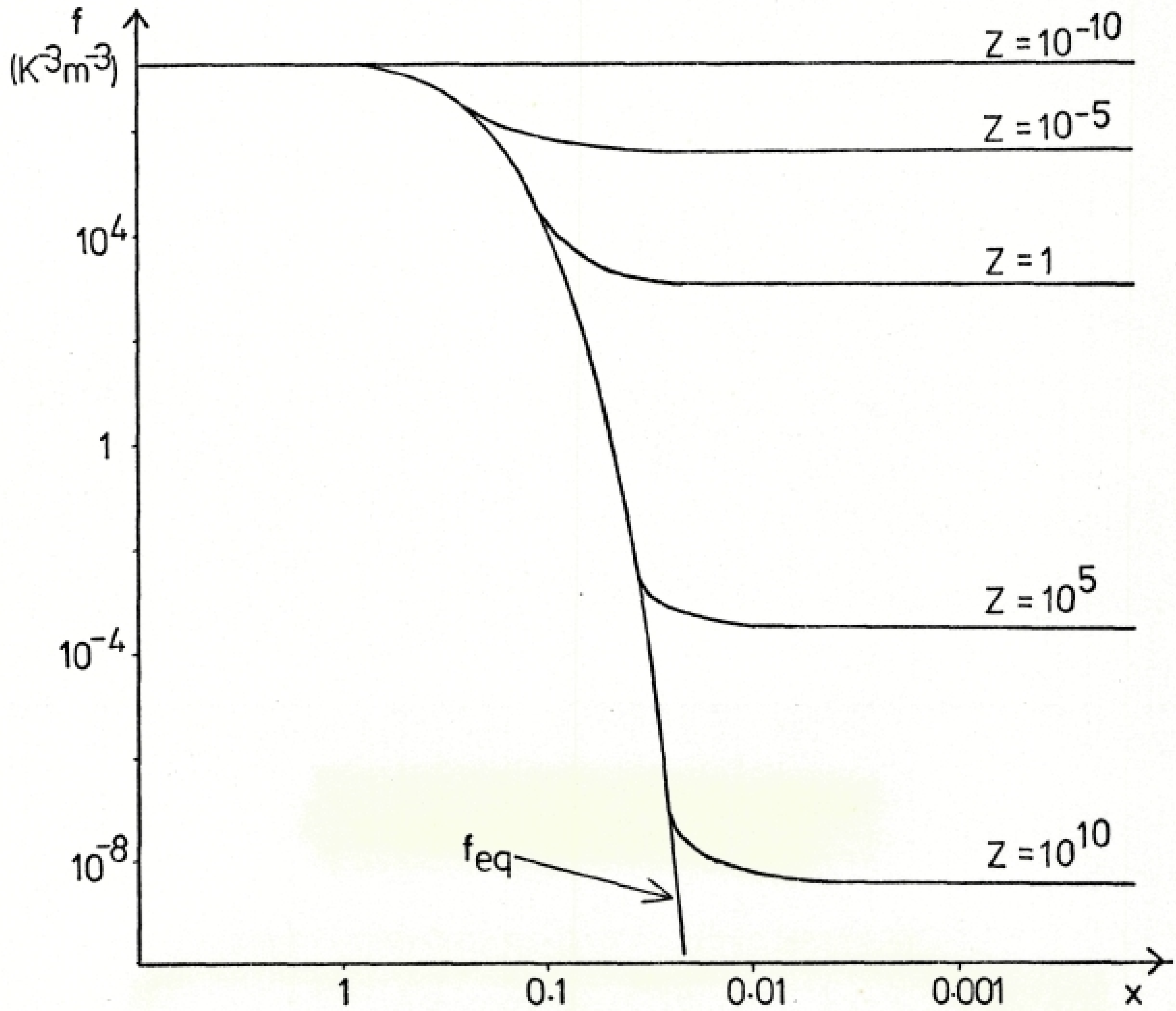
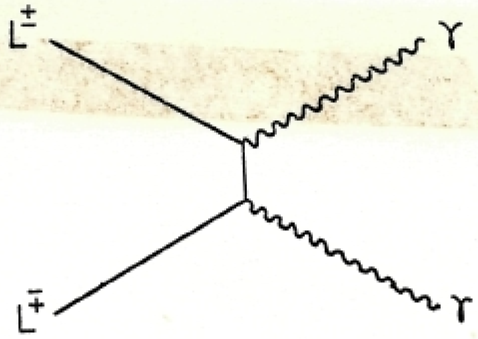
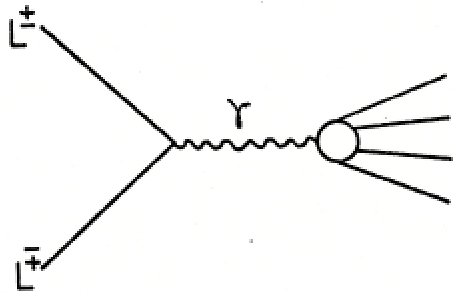


FIGURE 2



(a)



(b)

FIGURE 3

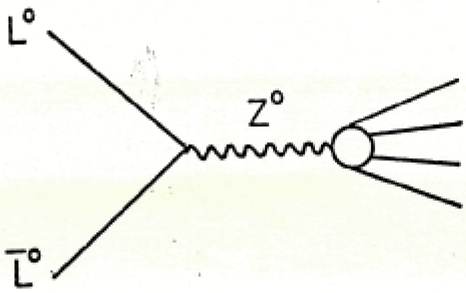


FIGURE 4

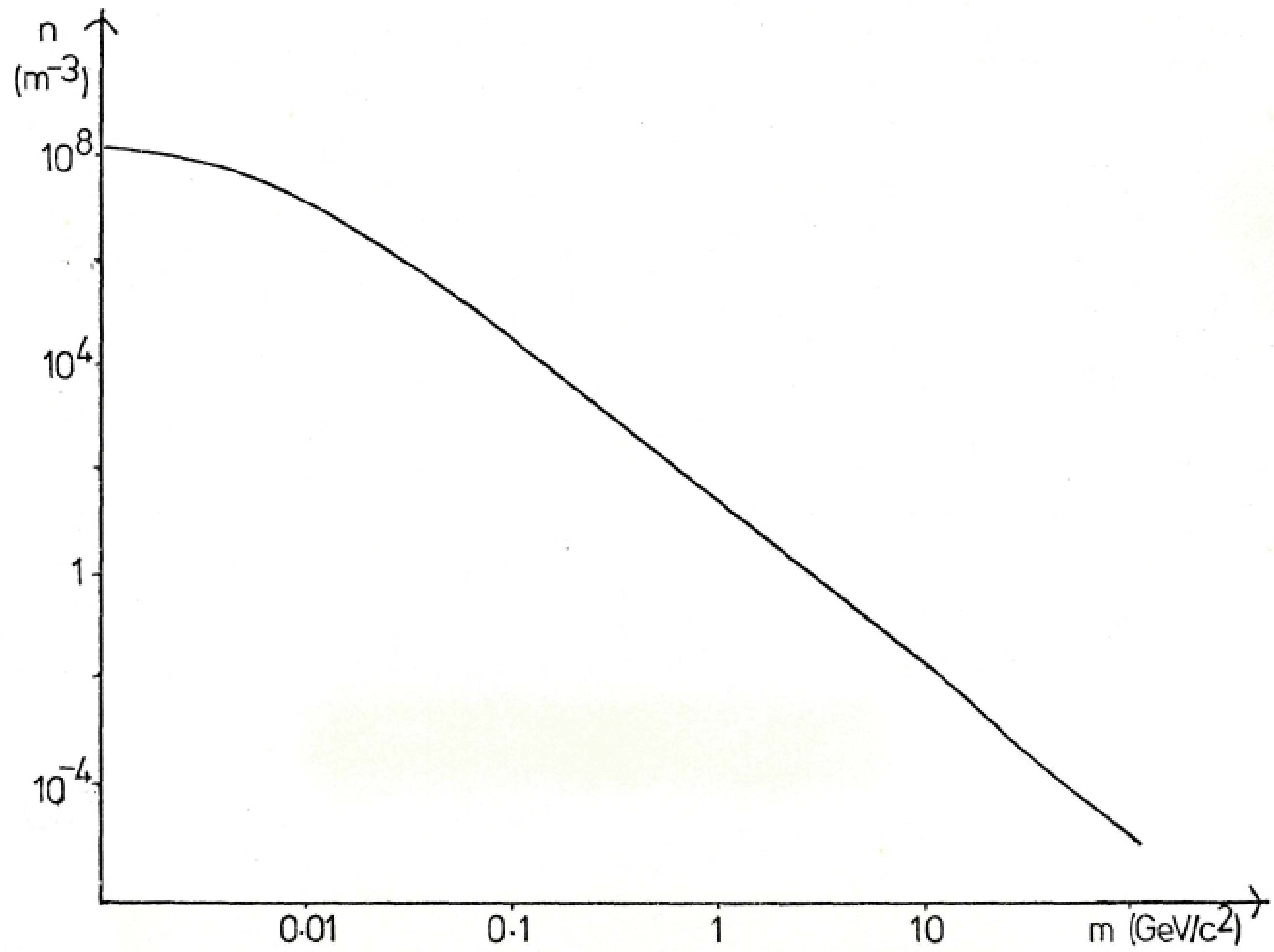


FIGURE 5

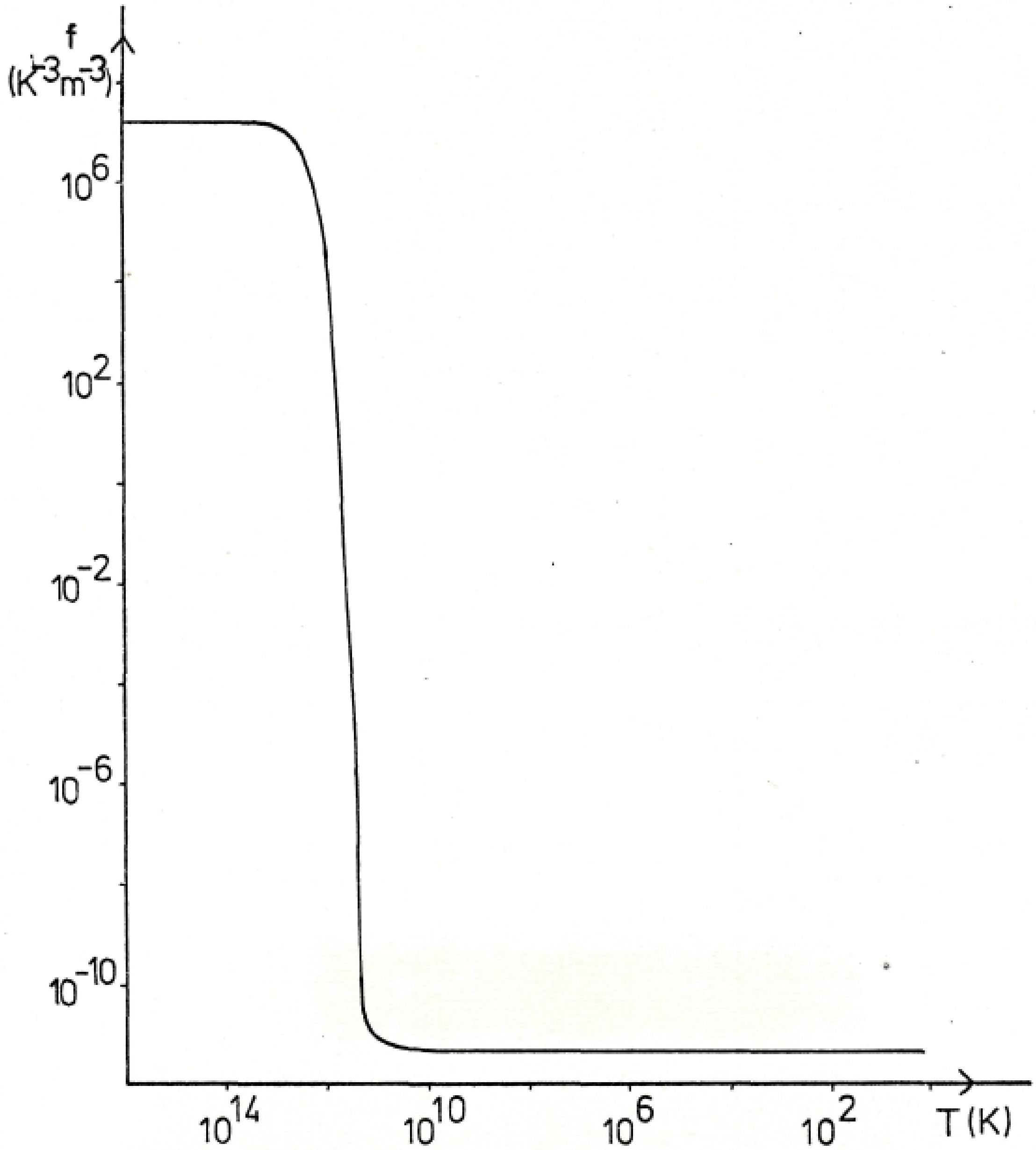


FIGURE 6

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Early Universe 1

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