BOUNDS ON PARTICLE MASSES IN THE WEINBERG–SALAM MODEL ☆

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Received 30 November 1978

Various conditions necessary for the self-consistency of the Weinberg–Salam model are used to place constraints on fermion and Higgs Boson masses. We find that spontaneous symmetry breakdown cannot generate fermion masses in excess of about 300 GeV.

In the Weinberg–Salam $SU(2)_L \times U(1)$ model for weak interactions, the masses of all the gauge bosons, quarks and leptons are taken to arise from the Higgs mechanism. At the tree approximation, the couplings of the Higgs scalar field ϕ to itself determine the effective potential $V(\phi)$, which in turn determines the symmetry of the "vacuum". In this approximation $V(\phi)$ is independent of the couplings (which determine the masses attained after spontaneous symmetry breakdown) of fermions and gauge bosons to ϕ. If, however, one-loop corrections to $V(\phi)$ are included, then the gauge bosons and fermions will influence $V(\phi)$. The requirement that this influence should not serve to prevent the possibility of spontaneous symmetry breakdown places several constraints on the couplings in the theory, and hence on the ratios of masses of various particles. Linde and Weinberg [1] have derived a lower bound on the mass of the Higgs particle H by demanding that the energy density of the "vacuum" after spontaneous symmetry breakdown should not exceed its value when $\phi = 0$. In this note, we apply the more complete requirement that the conventional "vacuum" in which $\langle\phi\rangle \neq 0$ corresponds to the absolute, rather than only a local, minimum of $V(\phi)$, at least in the domain where $V(\phi)$ may be obtained reliably from perturbation theory. If all fermion and gauge boson masses are generated from the vacuum expectation value of a single ϕ field, then this constraint allows one to place an upper bound on the fermion masses. The exact form of the bound involves m_W, m_H and other parameters, but typically the mass m_f of the heaviest fermion must satisfy $m_f \lesssim 300$ GeV. While this range is not immediately accessible to experimental investigation, the very existence of such a bound, coming solely from considerations of self-consistency, places constraints on models for weak interactions. Our bound is equivalent to an upper limit on the dimensionless fermion–Higgs Yukawa coupling, f, and it ensures that f is perturbatively small; $m_f \lesssim 300$ GeV corresponds to $f^2/4\pi \lesssim 0.1$.

In a theory with more than one coupling constant, one-loop graphs can dominate over tree graphs, while perturbation theory remains reliable because all couplings are small. For example, with a gauge coupling g and ϕ^4 self-coupling λ (both small), but such that λ is of order g^4, a gauge boson loop can compete with $O(\lambda)$ tree graphs, while yet higher-order corrections remain unimportant. However, even if the couplings are small, the perturbation expansion breaks down when logarithms of field strengths become large ‡1. In the following discussion, we shall simply require that the theory be consistent over the range of ϕ that can be explored perturbatively.

The complete formula for $V(\phi)$ in the one-loop approximation is [2]

$$V(\phi) = -\tfrac{1}{2}\mu^2\phi^2 + \tfrac{1}{4}\lambda\phi^4 + A\phi^4 \log(\phi^2/M^2)\,, \qquad \text{(1a)}$$

☆ Work supported in part by the U.S. Department of Energy under Contract No. EY76-C-03-0068.

[1] Alfred P. Sloan Foundation Fellow.

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‡1 Renormalization group improvement would be helpful only if the theory were asymptotically free.

where

$$A = \frac{1}{64\pi^2}\Big[\sum_{\text{gauge bosons}} 3g_i^4 - \sum_{\text{fermions}} f_i^4\Big], \qquad (1b)$$

and the g_i (f_i) are the couplings of the gauge bosons (fermions) to the Higgs particles. Note that, because of Fermi statistics, the fermion contribution to A is negative. The parameter M in eq. (1a) is a renormalization mass. In the Weinberg–Salam $SU(2)_L \times U(1)$ model, A is given by

$$A = \frac{1}{64\pi^2}\Big[3[2g^4 + (g^2 + g'^2)^2] \qquad (2)$$

$$-\left(\frac{g}{2}\right)^4 \sum_{\text{fermions}} \left(\frac{m_f}{m_W}\right)^4\Big], e = g\sin\theta_W = g'\cos\theta_W$$

In our numerical estimates, we use $\sin^2\theta_W = 0.23$, so that $m_W \approx 77$ GeV. We have dropped the $O(\lambda^2)$ contributions of Higgs scalar loops to $V(\phi)$, since, as discussed below, these must be negligible if perturbation theory is to be valid [‡2].

For spontaneous symmetry breakdown to occur it is necessary that $V(\phi)$ should have a non-trivial local minimum at $\phi = \phi_0$ such that

$$\phi_0 \neq 0, \quad \delta V/\delta\phi|_{\phi=\phi_0} = 0,$$

$$\delta^2 V/\delta\phi^2|_{\phi=\phi_0} = m_H^2 \geqslant 0\,. \qquad (3)$$

To investigate the consistency of a theory based on the "vacuum" $\phi = \phi_0$, we shall assume such a theory and then find under what circumstances inconsistencies appear. In that case, the parameters μ^2 and M^2 appearing in the effective potential $V(\phi)$ may be eliminated in favor of ϕ_0 and m_H. It is convenient to introduce

$$S \equiv \phi/\phi_0\,, \quad \Xi \equiv 4A\phi_0^2/m_H^2\,, \qquad (4a)$$

in terms of which

$$V(\phi) \equiv \tfrac{1}{8}m_H^2\phi_0^2\,\widetilde{V}(\phi) = \tfrac{1}{8}m_H^2\phi_0^2 S^2$$

$$\times [2\Xi S^2\log(S^2) - 3\Xi S^2 + 4\Xi + S^2 - 2]\,. \qquad (4b)$$

The requirement [1] $V(\phi_0) < V(0)$ necessary to allow spontaneous symmetry breakdown becomes

‡2 For quark loops, higher-order QCD corrections are governed by an effective coupling evaluated on the scale of ϕ_0 (see eq. (3)), and may therefore safely be ignored.

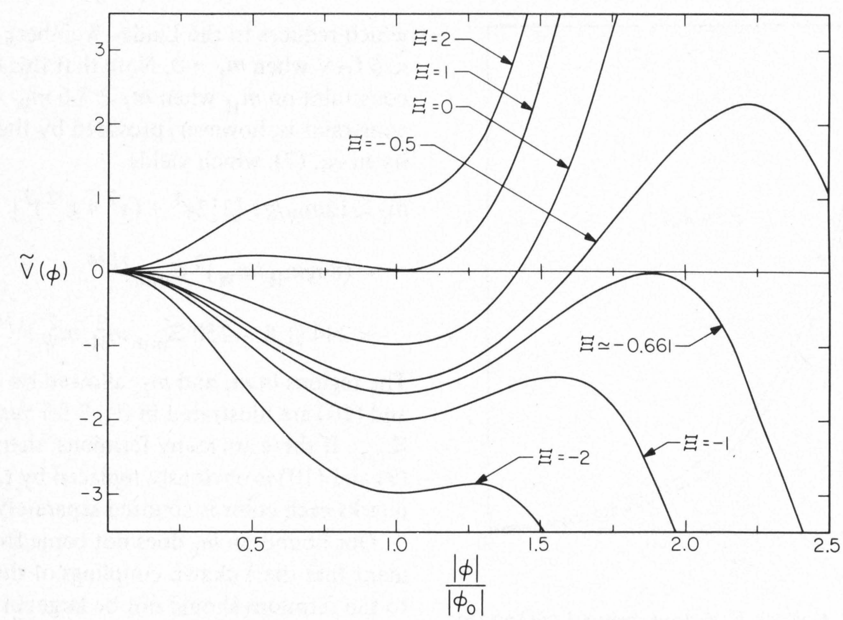

Fig. 1. The reduced effective potential $\widetilde{V}(\phi) = (8/m_H^2\phi_0^2)\,V(\phi)$ as a function of ϕ/ϕ_0 for various choices of the combination of couplings Ξ defined in eq. (4). For usual spontaneous symmetry breakdown to occur, $\phi = \phi_0$ must correspond to an absolute minimum of $V(\phi)$, at least within the range of ϕ accessible to perturbation theory.

$$V(\phi_0) = \tfrac{1}{8} m_{\mathrm{H}}^2 \phi_0^2 (\Xi - 1) < 0\,, \tag{5}$$

so that $\Xi < 1$.

In fig. 1 we plot $\widetilde{V}(\phi)$ as a function of ϕ/ϕ_0 for various values of Ξ. As the Yukawa couplings f_i increase, Ξ decreases, as does $V(\phi_0)$. For negative Ξ, a new phenomenon occurs: $V(\phi)$ eventually turns over and goes to $-\infty$ as $\phi \to \infty$. However, since our expression for $V(\phi)$ is obtained from perturbation theory, we have no estimate of it for values of ϕ so large that $A \log(\phi^2/\phi_0^2) \gtrsim 1$. We therefore do not consider its behavior as $\phi \to \infty$, but rather, require that $V(\phi) > V(\phi_0)$ for all values of $\phi \neq \phi_0$ within the range over which $V(\phi)$ is reliably calculated. If this is not satisfied, then the theory is inevitably inconsistent. Fig. 2 shows the values ϕ_1 of ϕ for which $V(\phi_1)$ becomes less than $V(\phi_0)$, as a function of Ξ. (We also show the values of ϕ corresponding to the second local maximum of $V(\phi)$.) For large values of $\log(\phi_1/\phi_0)$, one finds

$$\Xi \approx -[4 \log(\phi_1/\phi_0)]^{-1}\,. \tag{6}$$

If the theory is to allow a stable "vacuum" in perturbation theory then ϕ_1 must lie outside the range of validity of perturbative approximations. In practice, our

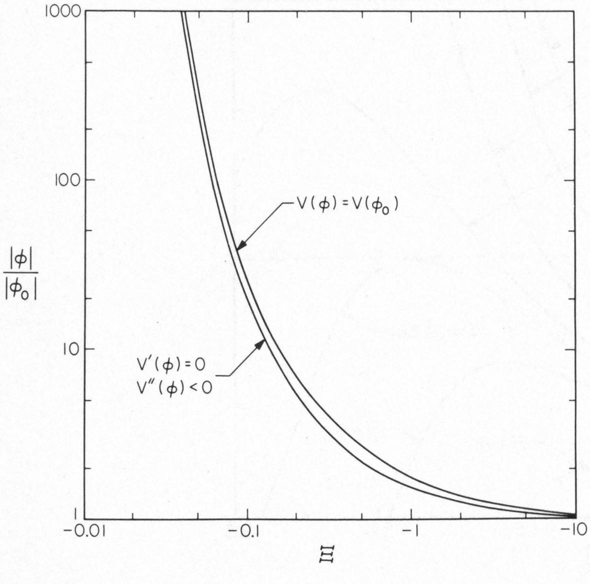

Fig. 2. The values of ϕ at which $V(\phi)$ drops below $V(\phi_0)$ and at which the second local maximum of $V(\phi)$ occurs, as a function of Ξ. These values of ϕ must be so large that our perturbative methods fail if the "vacuum" $\phi = \phi_0$ is to be stable.

final results are rather insensitive to the precise value of Ξ which is deemed unacceptable. Combining the Linde–Weinberg condition [1] with our requirements on $V(\phi)$ one obtains

$$1 > \Xi > -|\Xi_{\min}|\,, \tag{7}$$

where $|\Xi_{\min}|$ is presumably much less than 1 and perhaps as small as 0.005.

For any particular set of fields and couplings, one can translate these bounds on Ξ into bounds on ratios of particle masses. Consider first the case of the Weinberg–Salam $SU(2)_L \times U(1)$ model with its one complex $SU(2)_L$ doublet of Higgs fields and with a single heavy fermion. In this case (g and g' are defined in eq. (2))

$$\Xi = (m_{\mathrm{W}}^2/64\pi^2 m_{\mathrm{H}}^2 g^2)\ \{3[2g^4 + (g^2 + g'^2)^2] - (g m_{\mathrm{f}}/2m_{\mathrm{W}})^4\}\,. \tag{8}$$

The first inequality in eq. (7) then becomes

$$m_{\mathrm{H}} \gtrsim (m_{\mathrm{W}}/8\pi g)\ \{3[2g^4 + (g^2 + g'^2)^2] - (g m_{\mathrm{f}}/2m_{\mathrm{W}})^4\}^{1/2} \approx \{24[1.8 - 0.01\ (m_{\mathrm{f}}/m_{\mathrm{W}})^4]\}^{1/2}\ \mathrm{GeV}\,, \tag{9}$$

which reduces to the Linde–Weinberg bound [1] $m_{\mathrm{H}} \gtrsim 6$ GeV when $m_{\mathrm{f}} \to 0$. Note that this bound places no constraint on m_{H} when $m_{\mathrm{f}} \gtrsim 3.6\, m_{\mathrm{W}} \approx 280$ GeV. A constraint is, however, provided by the second inequality in eq. (7), which yields

$$m_{\mathrm{f}} \lesssim (2m_{\mathrm{W}}/g)\ \{3[2g^4 + (g^2 + g'^2)^2] - (8\pi g m_{\mathrm{H}}/m_{\mathrm{W}})^2\ \Xi_{\min}\}^{1/4} \approx 244\,\{1.8 - 250\ \Xi_{\min}\, m_{\mathrm{H}}^2/m_{\mathrm{W}}^2\}^{1/4}\,. \tag{10}$$

The regions in m_{f} and m_{H} allowed by the bounds (9) and (10) are illustrated in fig. 3 for various choices of $\Xi_{\min}$. If there are many fermions, then the m_{f} in eqs. (9) and (10) is obviously replaced by $(\Sigma_i\, m_{fi}^4)^{1/4}$; for quarks each color is counted separately.

Our bound on m_{f} does not come from the requirement that the Yukawa couplings of the Higgs bosons to the fermions should not be large; in fact, so long as $(m_{\mathrm{H}}/m)^{1/2}$ is not enormous it is much more stringent. However, for a perturbative investigation of the theory to be at all meaningful, it is necessary that higher and

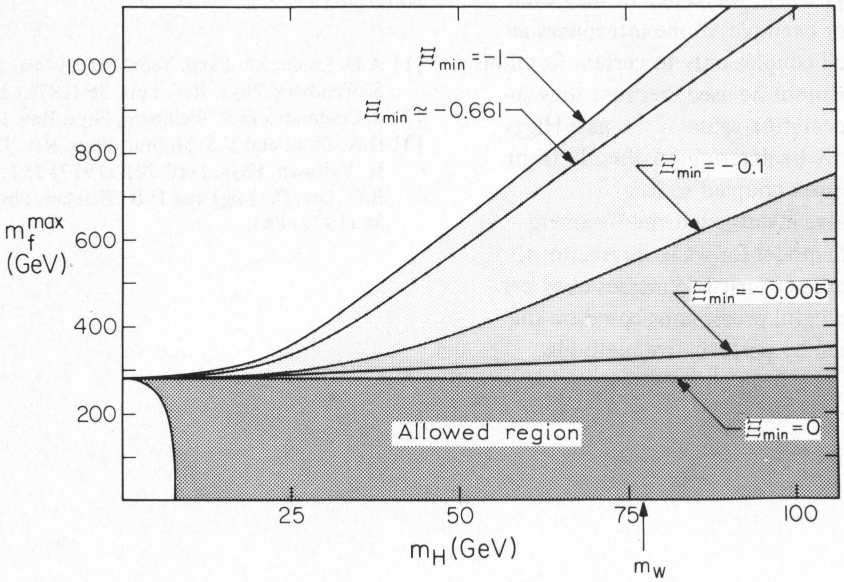

Fig. 3. The domains in the mass of the Higgs particle and of the heaviest fermion for which the Weinberg–Salam model is consistent. The value of Ξ_{min} depends on the region of validity of perturbation theory; Ξ_{min} is probably very small. The forbidden region in the lower left-hand corner represents the Linde–Weinberg bound.

higher orders in the perturbation series should give systematically smaller contributions. Experiments have shown that g and g' satisfy this condition, and our bounds on m_f ensure that it will hold for the f_i. The quartic self-couplings λ of the Higgs bosons must also obey the condition, so that ‡3

$$\lambda/4\pi^2 = g^2 m_H^2 (1 + \tfrac{16}{3}\,\Xi + O(\Xi^2))/16\pi^2 m^2 \ll 1 ,$$

or

$$m_H \ll 4\pi m_W/g \approx 1000 \text{ GeV} . \tag{11}$$

All predictions of the theory are obtained by perturbative methods, and, if the bound (11) were not satisfied then no predictions could be made ‡4.

We have given bounds on the Higgs particle mass (eqs. (9), (10) and (11)) which result from demanding consistency of the theory. However, by making the specific assumption that the term $\mu^2\phi^2$ in $V(\phi)$ vanishes ‡5, one may obtain a definite prediction for m_H [2]:

$$m_H \approx (m_W/4\sqrt{2}\pi)\{3[2g^4 + (g^2 + g'^2)^2] - \sum (g m_f/2m_W)^4\}^{1/2} . \tag{12}$$

If the fermion term can be ignored, then this gives $m_H \approx 9$ GeV – close to the range of present experiments.

In this paper, we have concentrated on the simplest workable model for weak interactions, since there is so far no compelling experimental evidence for a more complicated structure. In more complicated models our

‡3 For the purposes of computing higher-order corrections to the effective λ, we have defined $\lambda \equiv \frac{1}{3}\delta^4 V/\delta\phi^4 |_{\phi=\phi_0}$.

‡4 Similar conclusions have been reached by demanding that the high-energy interactions of Higgs particles in the Born approximation should not violate unitarity [3].

‡5 If dimensional regularization is used, then the ϕ^2 counterterms generated at each order in the perturbation series must be proportional to the bare μ^2, since the renormalization mass (which allows the coupling constant to attain dimensions away from $d = 4$) can enter only in logarithms. Hence the vanishing of the renormalized μ^2 in $V(\phi)$ which was suggested in ref. [2] may be preserved naturally to all orders, despite the fact that no symmetry requires it. It would naively be guaranteed by scale invariance, but this is violated by renormalization. Nevertheless, the violations in perturbation theory are logarithmic and do not provide a scale for the mass.

bounds may be strengthened, weakened or may even disappear entirely. For example, if one introduces an extra Higgs field which couples only to certain fermions, then our bounds (7) cannot be used, because they involve the vacuum expectation value of the new Higgs field, which would only be determined directly from the mass of a gauge boson coupled to it.

To conclude, we have investigated the Weinberg–Salam $SU(2)_L \times U(1)$ model for weak interactions, and find that unless ratios of particle masses obey certain bounds, no meaningful predictions based on the model may be obtained by perturbative methods.

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