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QCD Expectations for High-Energy Hadronic Collisions[1]

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Abstract. It is argued that perturbative QCD effects could be important in low-momentum transfer hadronic collisions at very high energies and should then give rise to several distinctive phenomena.

In processes such as deep-inelastic lepton-hadron scattering, e^+e^- annihilation and high transverse momentum hadron collisions, partons receive large momentum transfers, and may thus attain large invariant masses. Such off-shell partons dissipate their masses through gluon radiation, as described by QCD perturbation theory [1]. The average transverse momentum of final hadrons is proportional to the invariant mass of the original parton (as recently investigated in high-energy e^+e^- annihilation [2]).

Most hadron-hadron collisions at available c.m. energies ($\sqrt{s} \lesssim 60$ GeV), give final hadrons collimated within cylinders of fixed transverse momentum $\simeq 0.3$ GeV along the beam directions [3]. Here, we shall argue that at sufficiently high energies even hadronic collisions involving low momentum transfers may generate partons with large invariant masses. The resulting increase in final hadron transverse momenta would be observable in the next generation of $\bar{p}p$ colliding beam experiments [4] ($540 \lesssim \sqrt{s} \lesssim 800$ GeV).

We shall discuss the qualitative features of high-energy hadronic collisions in the context of a simple model. Some justification of the various assumptions in this model is given below. To a first approximation, we take the incoming hadrons to consist only of "valence" quarks carrying fixed fractions of the total hadron c.m. energy $E = \sqrt{s}/2$. The interaction is assumed to occur by exchange of a small average four-momentum Δ_μ between quarks from the two hadrons.* (The detailed mechanism for the momentum transfer is not considered: we consider merely an "effective interaction.") The acceleration resulting from this momentum transfer forces the quarks to attain invariant masses $\sim \sqrt{E\Delta}$, thus causing the quarks to emit gluon radiation. So long as Δ_μ does not decrease as E increases, such invariant masses should eventually become large and lead to observable effects. This phenomenon is directly analogous to the emission of Bremsstrahlung by high energy charged particles traversing matter [5]: the charged particles receive small deflections $\sim \Delta$ from Coulomb interactions with nuclei (which occur roughly every radiation length) and thus attain invariant masses $\sim \sqrt{E\Delta}$ to be dissipated by photon emission.

The momentum transfer Δ may be determined empirically by a direct but approximate method. Let the c.m.s. four-momenta of the initial hadrons be p_1 and p_2, so that the momenta of the final hadronic systems become $p_1 + \Delta$ and $p_2 - \Delta$. For fixed $|\Delta_\mu| \ll E$, the invariant masses of these systems become $M^2 \sim 2p \cdot \Delta$, and should thus increase roughly linearly with $\sqrt{s}$. An experimental measurement of the total invariant mass of hadrons in each of the two hemispheres centered on the incoming beam directions could thus be used to deduce the value of Δ. If final hadrons are produced uniformly in the available rapidity range, then one may estimate**

* This is similar to Feynman's assumption about an exchange of "wee" four-momentum in non-diffractive inelastic hadronic collisions. Note in particular that Feynman's wee exchange has non-zero longitudinal component [11]

** If the total available rapidity interval is $Y \sim \log(s/m^2)$, then the total longitudinal momentum in the hemisphere $y > 0$ is roughly $P_{\|}^{\text{tot}} = \int d\sigma/(d^2P_\perp dy) P_\perp d^2P_\perp \left[\cosh\left(\frac{Y}{2}\right) - 1\right]$, while the energy $E^{\text{tot}} = \int d\sigma/(d^2P_\perp dy) P_\perp d^2P_\perp \sinh\left(\frac{Y}{2}\right) \simeq \sqrt{s}/2$ (the total transverse momentum vanishes by definition). Then $M^2 = [E^{\text{tot}}]^2 - [P_{\|}^{\text{tot}}]^2 \simeq \sqrt{s}\int d\sigma/(d^2P_\perp dy) P_\perp d^2P_\perp = H\langle |P_\perp| \rangle \sqrt{s}$.

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$M^2 \sim H\langle|P_\perp|\rangle\sqrt{s}$, where H is the average number of hadrons produced per unit rapidity interval: the approximate measured constancy of H [6] and $\langle|P_\perp|\rangle$ for $10 \lesssim \sqrt{s} \lesssim 60$ GeV thus supports the assumption of a fixed Δ_μ, with magnitude $|\Delta_\mu| \sim 1$ GeV.

The distribution of momentum transfers Δ is determined qualitatively by the Fourier transform of the quark–quark interaction potential. The very short distance Coulomb-like part of this potential is responsible for high transverse momentum hadron production, but does not contribute significantly to the total hadronic scattering cross-section. At distances larger than the radii of the incident hadrons, QCD interactions are presumably shielded, so that the effective quark–quark potential goes to zero; the Δ distribution should thus be damped when any component of Δ becomes smaller than the fixed inverse hadron radius. The dominant scattering should occur with intermediate impact parameters, and give rise to a Δ distribution peaked at small values.* In the following, we make the crucial assumption that the quark–quark scattering cross-section has $\langle|\Delta_\mu|\rangle$ for each component μ fixed and non-zero at high energies (see also the first footnote of the paper).

An important assumption of our model is that scattering should occur by independent and incoherent interaction of one quark from each hadron. (This "additivity" assumption formed the basis of several successful previous phenomenological investigations of the dynamical quark model [7]. The rather direct evidence for "additivity" from comparisons of total hadron scattering cross-sections is discussed below.) The basic criterion for the validity of the assumption is that the range of the scattering interaction should be smaller than the average distance between quarks in the initial hadrons. With the phenomenological result $\Delta \sim 1$ GeV, the range would be ~ 0.2 fm, while typical light hadron radii are $\sim 0.7 - 1.0$ fm, so that the condition is at least not grossly violated**.

We now estimate the invariant masses μ_i^2 attained by incoming or outgoing quarks in the $2 \to 2$ scattering process of Fig. 1. As discussed above we assume that all four components of the momentum transfer Δ in the c.m. frame are of fixed magnitude, independent of $|\mathbf{P}|$. In the limit that the c.m.s. momenta of the incoming quarks $|\mathbf{P}| \gg |\Delta|$, this assumption

* For Example, an effective interquark potential $V(r) \sim \Lambda^2 r$ would give a differential cross-section $\sim \Lambda^2/\Delta^4$

** A similar criterion governs the applicability of independent nucleon models for medium energy hadron–nucleus and nucleus–nucleus interactions. For collision energies above a few hundred MeV, such models are found experimentally to be quite accurate [8]. Since the average momentum transferred (Δ) in low-energy NN collisions is also 0(100 MeV), collision energies of only a few times Δ are required for interactions to be approximately independent

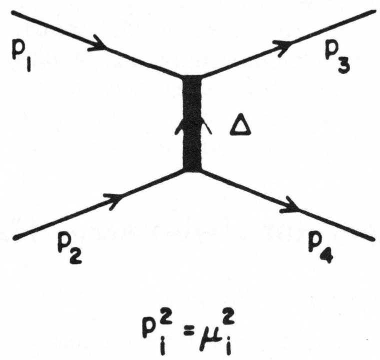

Fig. 1. Momentum assignments for two-body scattering of off-shell particles

yields a definite relation between the components of Δ and the μ_i^2 ($\|$ denotes component along $\mathbf{P}$):

$$\Delta_\| \simeq \frac{1}{4|\mathbf{P}|}[(\mu_3^2 + \mu_4^2) - (\mu_1^2 + \mu_2^2)]$$

$$\Delta_0 = \frac{1}{4|\mathbf{P}|}[(\mu_3^2 - \mu_1^2) + (\mu_2^2 - \mu_4^2)]$$

$$\Delta^2 \simeq \frac{1}{4|\mathbf{P}|^2}[-\mu_1^2\mu_2^2 + \mu_2^2\mu_3^2 - \mu_3^2\mu_4^2 + \mu_4^2\mu_1^2]. \qquad (1)$$

For large $|\mathbf{P}|$, rest masses of any of the quarks are negligible. Equation (1) demonstrates that even if no energy were transferred ($\Delta_0 = 0$)*, transfer of momentum parallel to $\mathbf{P}$ is sufficient to effect the acceleration necessary to generate $\mu_i^2 \neq 0$. Note that radiation of real or timelike invariant mass particles from initial or final quarks must give $\mu_1^2, \mu_2^2 \leqq 0$ and $\mu_3^2, \mu_4^2 \geqq 0$. Typically, the average invariant mass attained by incoming or outgoing quarks is $\langle|\mu_i^2|\rangle \sim |\Delta_\||\,|\mathbf{P}|$.

The kinematics of low-momentum transfer two-body scattering discussed in the previous paragraph should be contrasted with those for the case of large momentum transfers. When the transverse or longitudinal momentum exchanged is not fixed, but instead increases with $|\mathbf{P}|$, the initial and final quarks may have a range of invariant masses from zero up to $\sim |\mathbf{P}|^2$. The quarks may thus remain on their mass shells only if either $\Delta_\| = 0$** or some component of Δ increases as $\sim\sqrt{|\mathbf{P}|}$ or $\sim |\mathbf{P}|$.

Approximating incoming protons by three valence quarks, the c.m.s. momentum in the qq collision is roughly $|\mathbf{P}| \sim \sqrt{s}/6$. Hence, at ISR energies $\sqrt{s} \sim 60$ GeV, we expect an average "struck" quark invariant mass $\mu \sim 3$ GeV while at $\sqrt{s} \sim 600$ GeV, $\mu \sim 10$ GeV, assuming $|\Delta_\mu| \sim 1$ GeV.

The final state of a high-energy hadronic collision

* As in potential scattering

** This is the case for the elastic scattering at high energy and fixed Δ^2 ($\Delta_\| \to \Delta/P$). It is clear that fixed Δ^2 does not necessarily imply fixed $\Delta_\|$. Our crucial dynamical assumption is the Feynman's wee *four*-momentum exchange should contain two basic components: one from the "struck" quarks and one from the "spectator" quarks in the original hadrons. The hadronic systems arising from the "spectators" should exhibit a fixed transverse spread (governed by the inverse size, and hence "Fermi motion" in the original hadrons) independent of the collision energy. On the other hand, the discussion above indicates that the "struck" quarks should carry larger invariant masses as $\sqrt{s}$ increases, and thus give rise to progressively broader hadron jets. For very large "struck quark" invariant masses, these hadron jets should exhibit a definite "forked" structure: their invariant masses are dissipated by a high transverse momentum $q^* \to qG$ emission, followed by a sequence of much smaller transverse momentum radiations [1]. For smaller "struck quark" invariant masses, the transverse momentum ordering of successive emissions is less strong, and the final hadron jet should become broader but remain azimuthally symmetrical, exhibiting no definite substructure. The transverse momentum between "subjets" in an event involving small momentum transfer is bounded by the invariant mass $\sim \sqrt{|\mathbf{P}|\Delta}$ attained by the "struck" quark. To achieve higher transverse momenta, a large momentum must be transferred in the basic parton scattering process.*

The structure of hadron jets resulting from the "struck" quark in low momentum transfer hadronic collisions may be estimated by comparison with jets produced in other processes by quarks with similar invariant masses**. The most convenient such other process is e^+e^- annihilation at high c.m.s. energy Q. The $q\bar{q}$ produced by the γ^* decay in this case have a distribution of invariant masses extending up to $\mu^2 \sim Q^2$. For very large Q, this distribution should take on a "double log" form, giving an average invariant mass $\langle \mu^2 \rangle \sim (2\alpha_s(Q^2)/3\pi)Q^2$. At finite Q, both kinematic and hadronic effects modify this form: explicit Monte Carlo studies [1]*** nevertheless yield numerically similar results for $Q \gtrsim 10$ GeV. In e^+e^- annihilation, two virtual quark "legs" may radiate; in quark–quark scattering, the two initial as well as the two final "legs" may radiate, giving in total roughly twice the intensity of radiation. A rough estimate then suggests that the "struck" quark jet in hadronic collisions with qq subprocess c.m. energies $\sqrt{\hat{s}}$ should have a structure similar to that of an e^+e^- annihilation event at $Q^2 \sim 4|\Delta|\sqrt{\hat{s}}/\alpha_s(Q^2)$ with one of its jets rotated so that its axis coincides with the other jet.

Using this approximate correspondence with e^+e^- annihilation, one may estimate the structure of hadronic collisions at very high energies. For $pp(\bar{p})$ collisions $\sqrt{\hat{s}} \sim \sqrt{s}/3$ (so long as $\log\log(s/\Lambda^2)$ is not too large). The average transverse momentum of hadrons produced in such collisions is expected to remain roughly constant until $\sqrt{s} = 0(100\text{ GeV})$; it should then rise in correspondence with its behavior in e^+e^- annihilation. One may estimate that $\langle P_\perp^2 \rangle$ should rise by a factor ~ 2 between $\sqrt{s} \sim 60$ GeV and $\sqrt{s} \sim 600$ GeV [1, 2]. Similarly, the longitudinal hadron momentum distributions should soften as $\sqrt{s}$ increases: violations of Feynman scaling should occur as the "struck" quark invariant mass increases in the same manner as in deep inelastic scattering or e^+e^- annihilation. The rise in hadron density at low rapidity observed in ISR experiments [3] should thus continue in high energy events: no "plateau" in rapidity should exist. Correspondingly, the total hadron multiplicity should increase faster than logarithmically with s, as indicated by rescaling of present e^+e^- annihilation data. Asymptotically, the hadron multiplicity should rise [9] $\sim \exp[\sqrt{\log(|P||\Delta|/\mu_0^2)}]$: slower than the $\sim \exp[\sqrt{\log(Q^2/\mu_0^2)}]$ rise expected in e^+e^- annihilation, but faster than any power of $\log(s/\mu_0^2)$*.

The kinematic constraint of (1) should give correlations between the invariant masses of the "struck" quarks thus yielding "long range" s-dependent correlations between hadron multiplicities in the two hemispheres.

Our simple model also has consequences for the total hadronic scattering cross-section. Above, we have implicitly assumed that incoming hadrons contain only their "valence" quarks (as apparent in spectroscopy). These quarks are taken to scatter incoherently and independently. If further total effective cross-sections for the quark–quark interactions are taken to be independent of the incoming quark energies (as if, for example, the cross-section is determined by the fixed hadron radius), then the observed s independence of $\sigma(pp)$ and the well-known "additive quark model" result [7] $\sigma(pp)/\sigma(\pi p) \sim 3/2$ are reproduced. The possibility of gluon radiation from the incoming "valence" quarks** gives corrections to these results. Processes

* The resulting events exhibit a distinctively different structure. For high transverse momenta, they usually involve four final hadron jets, while similar low-momentum transfer events would involve six jets

** The leading log approximation to perturbative QCD implies that gluon radiation from quarks with invariant masses $\gg \Lambda \sim 1$ GeV should be independent of the process by which the quark was produced

*** The invariant masses of jets may also be estimated from measured multiplicities and $P_\perp$ distributions (e.g., with respect to sphericity axis) by the method used above for hadronic collisions

* A slightly more rapid rise in hadron multiplicity is expected in πp compared to pp collisions since the average $\sqrt{\hat{s}}$ should be larger (inasmuch as the dominant constituents of the incoming hadrons are "valence" quarks)

** At large distances from their point of interaction, the incoming hadrons may plausibly be taken to contain essentially only "valence" quarks. At short times before the interaction, these quarks may radiate and thereby provide "sea" constituents in the hadrons. (The intensity of radiation and thus momentum fraction carried by the "sea" depends on the invariant mass which may be sustained by the interaction.)

such as $q \to qG^*$ may occur, in which the virtual gluon G^* has a spacelike invariant mass and absorbs the momentum transfer Δ to become on-mass shell, providing an additional contribution to the total cross-section. (This conclusion relies on the assumption discussed above that scatterings involving different incoming partons are entirely incoherent.) The magnitude of this contribution is determined by the multiplicity of gluons carrying a fraction x of the original valence quark momentum, and having (spacelike) invariant masses up to $|\mu^2| \sim |\Delta|\sqrt{x}|\mathbf{P}|$. The correction to the total cross-section is thus given in terms of the "gluon distribution" $G(x, \mu^2)$ and the intrinsic parton scattering cross-section σ_0 (assumed independent of energy and parton type) by $\Delta\sigma \sim \sim \sigma_0 \int_{G(x,|\mathbf{P}|\sqrt{x}|\Delta|)}^{1} G(x, |\mathbf{P}||\Delta|x)\,dx$ (with μ_0 a cutoff invariant mass $O(1\,\text{GeV})$). A very rough estimate indicates that this effect could account for the observed rise in the pp total cross-section at ISR energies, and indicates a further rise by a factor ~ 2–3 at $\sqrt{s} \simeq 600$ GeV.

The presence of gluons radiated from the incoming quarks was considered above to result in an increased hadron multiplicity. Kinematical constraints prevent most such gluons from contributing to increases in the total cross-section: the gluon invariant masses must not only be spacelike (so as to allow the radiating quark to remain on-shell) but must also obey the bound $|\mu^2| \lesssim |\Delta|\sqrt{x}|\mathbf{P}|$.

Events arising from scattering of the gluons radiated from the initial quarks (which presumably yield an increase in the total cross-section) should have final states with a distinct structure. Typically, the invariant of the "struck" parton (and hence the average hadron transverse momentum and multiplicity) in at least one hemisphere should be smaller than for events due to valence quark scattering.

The effective quark–quark interaction cross-sections depends on the initial hadrons in which the quarks are contained. The presence of heavy quarks reduces the hadron "size" or "color neutralization radius" according roughly to the reduced mass of the system, and hence reduces the maximum impact parameter at which scattering occurs, leading to an increased average momentum transfer Δ and a decreased $\sigma_0(\sim 1/\Delta^2)$ [10]. Such a decrease is indicated by comparison of $\sigma(\pi p)$ and $\sigma(\kappa p)$ or $\sigma(pp), \sigma(\phi p)$, $\sigma(\psi p)$, and also probably of $\sigma(\Sigma p)$ and $\sigma(pp)$ or $\sigma(p\bar{p})$. (Note also that $q\bar{q} \to GG$ annihilation may occur in addition to $qq \to qq$ or $q\bar{q} \to q\bar{q}$ scattering in $\pi^- p$, $\kappa^- p$ and $\bar{p}p$ collisions, but presumably becomes unimportant as s increases.)

In summary, we argue that low momentum transfer hadronic collisions may generate partons with large virtual masses $\mu^2 \sim \sqrt{s}$. There is a simple model of hadronic interaction based on the Feynman's wee longitudinal momentum exchange [11] which makes the conjecture plausible. Several distinctive phenomena are predicted for the pp collider energies.

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