HANDS-ON START TO WOLFRAM|ALPHA NOTEBOOK EDITION™

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NOTEBOOK EDITION™

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CHAPTER 12
Algebra in Wolfram|Alpha Notebook Edition

Introduction

The notebook environment in Wolfram|Alpha Notebook Edition is useful for solving equations, changing the form of equations and graphing equations, all in one unified document to explore concepts in algebra. Step-by-step solutions are also useful to check mechanics when solving problems in algebra to get real-time feedback. Many types of calculations and visualizations are possible in the software, and this chapter will outline a useful subset to act as a guide for exploring concepts in algebra.

Basic Algebraic Operations

Calculations in a notebook will automatically simplify algebraic expressions, and if a calculation contains a factored polynomial, the result will be the same factored form of the equation.

```
(a + 5)(a + 9)
step-by-step solution    full Wolfram|Alpha results
(a + 5)*(a + 9)
```

$(a+5)(a+9)$

To expand an equation into polynomial form, the term "expand" can be used.

```
expand (a + 5)(a + 9)
step-by-step solution    full Wolfram|Alpha results
Expand[(a + 5)*(a + 9)]
```

$a^2 + 14a + 45$

An equation can include symbols instead of integers for a calculation to expand an equation into polynomial form.

```
expand (a + b)(a + c)
Expand[(a + b)*(a + c)]
```

$a^2 + a\,b + a\,c + b\,c$

> While the characters *a*, *b*, *c*, *d*, *x* and *y* are commonly used to outline algebraic concepts, the software will accept any character as a symbol in algebraic calculations. The **Special Characters** menu also provides additional Greek characters that can be used as symbols in equations.

The **step-by-step solution** button creates a new pair of input and output cells to show the mechanics for expanding this equation. In this case, the solution involves the FOIL method, and step-by-step solutions follow a typical textbook explanation for solving this type of problem. Step-by-step solutions can be displayed one step at a time, or all of the steps can be displayed at one time.

```
show steps Expand[(a + b) * (a + c)]
```

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Results:

STEP 1

Expand the following:

$(a + b)\,(a + c)$

Show next step | Show all steps

A polynomial using symbols instead of numbers can also be computed into factored form using the term "factor" in the input cell.

factor a^2 + ab + ac + bc

```
Factor[a^2 + a*b + a*c + b*c]
```

$(a + b)(a + c)$

The result uses a space to represent multiplication. In calculations, either an asterisk or space can be used to represent multiplication. But in the calculation above, Wolfram|Alpha Notebook Edition makes the assumption that "ab" is "a" times "b" even without the space.

The term "together" can be used to put terms in a sum over a common denominator.

together 1/(x + 1) + 1/(x − 1)

```
Together[1/(x + 1) + 1/(x - 1)]
```

$$\frac{2x}{(-1 + x)(1 + x)}$$

Conversely, the term "apart" can be used to rewrite an equation as a sum of terms with minimal denominators.

apart 2x / ((−1 + x)(1 + x))

```
Apart[2*(x/((-1 + x)*(1 + x)))]
```

$$\frac{1}{x + 1} + \frac{1}{x - 1}$$

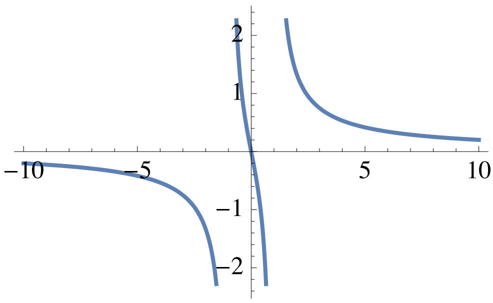
Calculations in Wolfram|Alpha Notebook Edition can include extra spaces as well, which is sometimes a handy way to make a complicated equation easier to read. In the calculations above, an extra space is added to separate the terms and the calculation still returns the expected result.

When working with equations and different forms of an equation, creating various graphs can often give a useful perspective. The term "that" can be used in any calculation to reference the previous result, and in this case creates a graph of the equation in the previous result.

graph that, where x goes from –10 to 10

related computations full Wolfram|Alpha results

```
Plot[(-1 + x)^(-1) + (1 + x)^(-1), {x, -10, 10}]
```



Calculations can also collect terms to factor common terms out.

collect ax^2 + bx^2y + cxy, x

```
Collect[a*x^2 + b*x^2*y + c*x*y, x]
```

$c\,x\,y + x^2\,(a + b\,y)$

Radicals

Radicals, like square roots, can be described in everyday English in calculations.

square root of x^2

```
Sqrt[x^2]
```

$\sqrt{x^2}$

In addition to using everyday English, the **Special Characters** menu at the upper-right corner of a notebook can be used to enter a square root symbol that more closely mirrors textbook notation. This formatting is often preferred to make the calculation more concise.

√(x^2)

Sqrt[x ^ 2]

$\sqrt{x^2}$

The result above demonstrates that Wolfram|Alpha Notebook Edition can solve problems across many levels of math, including complex numbers. In introductory courses, it might be useful to add an assumption that x is greater than 0 to the calculation to simplify the radial.

simplify √(x^2), x > 0

x

x

Instead of using a variable, a calculation can involve specific numbers within radicals, including a cube root.

∛3

alternate interpretations

step-by-step solution

full Wolfram|Alpha results

Assuming "∛" is the real–valued root | Use the principal root instead

CubeRoot[3]

$\sqrt[3]{3}$

Wolfram|Alpha Notebook Edition always provides results in exact form, if possible. The term "that" can be used to reference the previous result in a new calculation to calculate a decimal approximation.

approximate that

N[%]

1.44225

The calculation above used an integer; when a calculation starts with a decimal approximation, the calculation is no longer exact, and a decimal approximation is returned in the result. This overall convention can be used to calculate a result in decimal format for a wide variety of calculations.

$\sqrt[3]{3.0}$

alternate interpretations

full Wolfram|Alpha results

Assuming "$\sqrt[3]{}$" is the real–valued root | Use the principal root instead

```
CubeRoot[3.]
```

1.44225

The term "simplify" can be used to simplify expressions that contain radicals.

simplify $\sqrt{16x}$

```
Simplify[Sqrt[16*x]]
```

$4\sqrt{x}$

Simplification of an expression can be performed with specific numerical values rather than variables.

Use of parentheses is sometimes useful when a calculation involves a square root. For example, $\sqrt{16} + x$ and $\sqrt{(16 + x)}$ are interpreted differently; the first represents taking the square root of 16 then adding x, while the second represents the square root of $16 + x$.

simplify $\sqrt{(15/16)}$

```
Simplify[Sqrt[15/16]]
```

$$\frac{\sqrt{15}}{4}$$

The **step-by-step solution** button for the previous calculation creates a new pair of input and output cells to show the steps for this simplification. Earlier in this chapter, a step-by-step solution was shown to expand an equation, and the steps to simplify an expression with a radial are much different. Step-by-step solutions in the software contain a large variety of solutions and automatically identify the type of problem to provide an explanation for the solution that mirrors a typical textbook.

```
show steps Simplify[Sqrt[15/16]]
```

Results:

STEP 1

Simplify the following:

$$\sqrt{\frac{15}{16}}$$

Show next step | Show all steps

For any supported step-by-step solution, the buttons are the same to work through the solution one step at a time or show all the steps at once. The steps themselves and the quantity of steps will differ with different problems, but the buttons and formatting will always look the same.

Absolute Value

Calculations using an absolute value can be entered using everyday English or more typical notation found in a textbook.

```
absolute value of –7 + 4
```

```
Abs[–7] + 4
```

11

The typeset notation for absolute value can be entered on a typical keyboard by pressing the Shift key with the \ key.

```
−|7| + 5 + |−3|
```

step-by-step solution   full Wolfram|Alpha results

```
−Abs[7] + 5 + Abs[−3]
```

1

Calculations can use variables with absolute values to simplify algebraic expressions.

```
|3a| + 5a − 6 + |2a|
```

step-by-step solution   full Wolfram|Alpha results

```
Abs[3*a] + 5*a − 6 + Abs[2*a]
```

$5|a| + 5a - 6$

The term "result" can be used to reference the previous result and substitute a value of -1 for the variable a in the equation.

```
result where a = −1
```

full Wolfram|Alpha results

```
−6 + 5*a + 5*Abs[a] /. {a −> −1}
```

−6

Both the terms "that" and "result" can be used to reference the previous calculation, and the user can choose the phrase that makes sense in the context of the calculations.

Equation Solving

The term "solve" can be used for a wide variety of problems involving solving equations with one or more variables. When solving an equation with one variable, it is not necessary to specify the variable using the phrase "for x," but either phrasing will produce the desired result.

solve 3x − 6 = 12

step-by-step solution   full Wolfram|Alpha results

```
Solve[3*x - 6 == 12, x]
```

$\{\{x \to 6\}\}$

solve 3x − 6 = 12 for x

step-by-step solution   full Wolfram|Alpha results

```
Solve[3*x - 6 == 12, x]
```

$\{\{x \to 6\}\}$

Equations involving radicals can be used when solving for a particular variable.

solve $3x - \sqrt{5} = 12$

```
Solve[3*x - Sqrt[5] == 12, x]
```

$$\left\{\left\{x \to \frac{1}{3}\left(12 + \sqrt{5}\right)\right\}\right\}$$

To approximate the result, the term "result" can be used to reference the previous result and round it to 10 digits.

result rounded to 10 digits

```
N[{{x -> (12 + Sqrt[5])/3}}, 10]
```

$\{\{x \to 4.745355992\}\}$

Results sometimes contain radicals as well to represent the solution as an exact value.

solve x^2 + 2x – 1 = 0 for x

step-by-step solution full Wolfram|Alpha results

```
Solve[x^2 + 2*x - 1 == 0, x]
```

$\{\{x \to -1-\sqrt{2}\}, \{x \to \sqrt{2}-1\}\}$

When the result contains more than one solution, the output cell separates the solutions with curly brackets. The term "result" still refers to the set of solutions rather than one or the other solution, and the solutions can be rounded to five digits with one calculation.

result rounded to 5 digits

```
N[{{x -> -1 - Sqrt[2]}, {x -> -1 + Sqrt[2]}}, 5]
```

$\{\{x \to -2.4142\}, \{x \to 0.41421\}\}$

When a calculation includes two equations with two variables, the systems of equations are solved for both variables.

solve 2x + 4y = 14 and x – y = 4

```
Solve[{2*x + 4*y == 14, x - y == 4}, {x, y}]
```

$\{\{x \to 5, y \to 1\}\}$

When the result is not an integer, by default an exact result will be given using fractions.

solve 3x + 5y = 15 and x – y = 3

step-by-step solution full Wolfram|Alpha results

```
Solve[{3*x + 5*y == 15, x - y == 3}, {x, y}]
```

$\left\{\left\{x \to \frac{15}{4}, y \to \frac{3}{4}\right\}\right\}$

Two equations with two variables can also be solved in terms of symbol *a* when that symbol is used in the equations.

solve x + a = 10 and x − 2y = 4

step-by-step solution   full Wolfram|Alpha results

```
Solve[{x + a == 10, x - 2*y == 4}, {x, y}]
```

$$\left\{\left\{x \to 10 - a, y \to \frac{6-a}{2}\right\}\right\}$$

In the previous calculation, even with three symbols, Wolfram|Alpha Notebook Edition recognizes that *x* and *y* are the most commonly used variables and solves for both *x* and *y*.

At times, one of the equations might not include both variables, and in this case, the solution is still provided for both *x* and *y*.

solve 3x + 5y = 15 and x = 3

step-by-step solution   full Wolfram|Alpha results

```
Solve[{3*x + 5*y == 15, x == 3}, {x, y}]
```

$$\left\{\left\{x \to 3, y \to \frac{6}{5}\right\}\right\}$$

Step-by-step solutions are a useful component in Wolfram|Alpha Notebook Edition, and step-by-step solutions are available for algebraic calculations like solving equations. After clicking the **step-by-step solution** button in the previous input cell, a new pair of input and output cells are displayed with that step-by-step solution.

show steps solve (3 x + 5 y = 15, x = 3) for x and y

full Wolfram|Alpha results

Results:

STEP 1

Solve for y:

$$5\,y + 9 = 15$$

Show next step   Show all steps

This chapter will often show only the first step of a step-by-step solution, but the progression of steps is unique based on each type of calculation. In this case, the various steps make sense for solving equations, and might involve a much longer series of steps compared to calculations earlier in the chapter involving order of operations.

> As mentioned in a previous chapter, step-by-step solutions are live calculations and do not draw from a predefined list of solved problems. So there are many, many possible variations in step-by-step solutions.

The software can solve equations that use a greater-than symbol instead of an equal sign.

solve −4(x + 3) > 16

full Wolfram|Alpha results

```
Reduce[-4*(x + 3) > 16, x]
```

$x < -7$

Wolfram|Alpha Notebook Edition contains a sophisticated solver for equations, and a single calculation can include multiple radicals or more intricate equations.

solve √(x – 2) = √2 – √x

```
Solve[Sqrt[x – 2] == Sqrt[2] – Sqrt[x], x]
```

$\{\{x \to 2\}\}$

The step-by-step solution to solve this equation provides unique steps to simplify the terms with a radical. While it is common to enter a calculation first and then click the **step-by-step solution** button to show steps, any new input cell can contain the phrase "show steps" directly to generate the step-by-step solution.

show steps solve (sqrt(x – 2) = sqrt(2) – sqrt(x)) for x

Results:

STEP 1

Solve for x:

$$\sqrt{x-2} = \sqrt{2} - \sqrt{x}$$

Show next step | Show all steps

There are seven steps outlined in this particular solution, including several to navigate and simplify the terms with radicals.

STEP 2

Hint: Eliminate the square root on the left hand side.

Raise both sides to the power of two:

$x-2=\left(\sqrt{2}-\sqrt{x}\right)^2$

STEP 3

Hint: Move everything to the left hand side.

Subtract $\left(\sqrt{2}-\sqrt{x}\right)^2$ from both sides:

$-2-\left(\sqrt{2}-\sqrt{x}\right)^2+x=0$

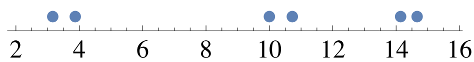
Graphing and Lines

Wolfram|Alpha Notebook Edition supports a wide variety of different types of graphs, as well as the ability to graph a wide variety of equations.

A number line can be used to visualize values for various numbers involving radicals.

√10, √15, √100, √115, √200, √215 on a number line

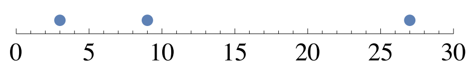
```
NumberLinePlot[{Sqrt[10], Sqrt[15], Sqrt[100], Sqrt[115], Sqrt[200],
    Sqrt[215]}]
```



Use of the phrase "number line" at the beginning of the calculations produces the same result as using that phrase at the end of a calculation. In addition, the phrase can even be shortened to just "line."

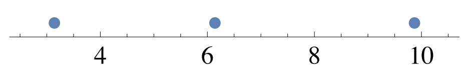
number line of 3, 3^2, 3^3

```
NumberLinePlot[{3, 3^2, 3^3}]
```



line π, π + 3, π^2

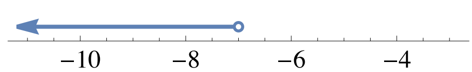
```
NumberLinePlot[{Pi, Pi + 3, Pi^2}]
```



In addition to a list of specific points, a number line can be used to visualize inequalities.

line −4(x + 3) > 16

```
NumberLinePlot[-4*(x + 3) > 16, x]
```

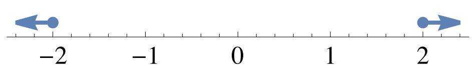


A number line can be used to graphically visualize solutions when solving an equation with inequalities. The first calculation provides the solution in algebraic form; the second provides the solution in graphical form.

```
solve x^2 + 3 − 5 >= 2
Reduce[x^2 + 3 − 5 >= 2, x]
```

$x \leq -2 \vee x \geq 2$

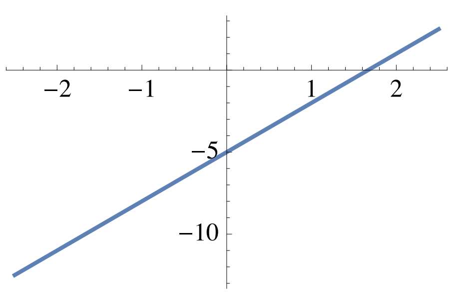
```
line x^2 + 3 − 5 >= 2
NumberLinePlot[x^2 + 3 − 5 >= 2, x]
```



Notice the number line has a solid dot when that point is included in the inequality (>=), and an open dot when that point is not included in the inequality (>).

The term "graph" is also a very flexible term to visualize a wide variety of equations, including equations in slope-intercept form.

```
graph y = 3x − 5
related computations
full Wolfram|Alpha results
Plot[3*x − 5, {x, −2.5, 2.5}]
```



In addition to creating a graph, the software includes knowledge about algebraic concepts and can be used to directly query the slope for a particular equation.

slope y = 3x – 5

alternate interpretations | full Wolfram|Alpha results

Assuming “slope” is referring to geometry | Use as referring to derivative computations instead

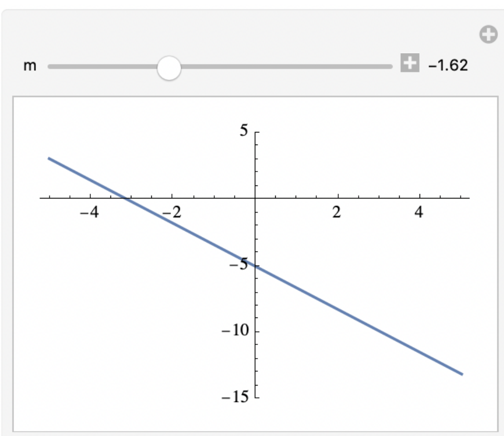
3

3

To visualize variations in slope, the variable *m* can be used to represent slope in a calculation that produces a graph, with *m* varying from –5 to 5. Use of the term "varying" creates a mouse-driven model to adjust the parameter and visualize the effects on the graph in real time. Other parameters for the *x* and *y* axes help to keep the view of the graph the same when moving the slider, which is often desirable for an animation.

graph y = m x – 5, x going from –5 to 5, y going from –15 to 5, varying m from –5 to 5

```
Manipulate[Plot[m*x – 5, {x, –5, 5}, PlotRange –> {–15, 5}],
  {{m, 0}, –5, 5, Appearance –> "Labeled"}]
```



Throughout this book, different variations of phrasing have been used to specify the domain and range for a graph (the values for the *x* and *y* axes). While the software often chooses useful values automatically, the graph above specifies values for both the *x* and *y* axes. This can be specified on any type of graph, not just this animation.

The *y* intercept is also a common concept to query in an equation, and can be queried directly.

y–intercept of y = 3x – 5

alternate interpretations | full Wolfram|Alpha results

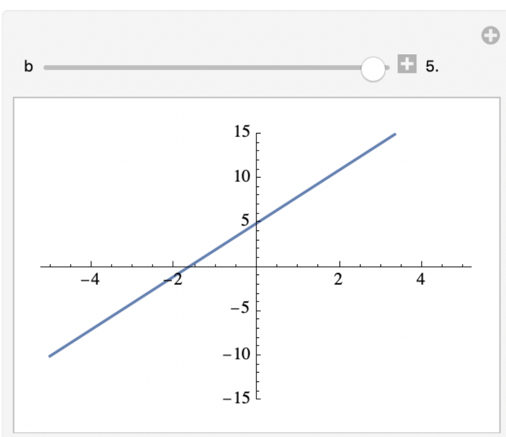
Assuming "y–intercept" is referring to geometry | Use "y" as a variable instead

–5

Similar to a previous example, use of the term "varying" with the symbol *b* changes that value and adjusts the graph in real time based on those changes to the equation. The slider shows variations from –5 to 5 to easily visualize where the line intercepts the *y* axis for many specific cases.

graph y = 3 x + b, x going from –5 to 5, y going from –15 to 15, varying b from –5 to 5

```
Manipulate[Plot[3*x + b, {x, -5, 5}, PlotRange -> {-15, 15}],
  {{b, 0}, -5, 5, Appearance -> "Labeled"}]
```



A calculation to determine the x intercept of an equation is possible as well since Wolfram|Alpha Notebook Edition understands this overall concept through its built-in knowledge. This solution can be compared to explicitly solving the equation where y equals 0 to reinforce this concept.

x–intercept of y = 3x – 5

alternate interpretations full Wolfram|Alpha results

Assuming "x–intercept" is referring to geometry | Use "x" as a variable instead

```
5/3
```

$$\frac{5}{3}$$

solve 3x – 5 = 0 for x

step-by-step solution full Wolfram|Alpha results

```
Solve[3*x - 5 == 0, x]
```

$$\left\{\left\{x \to \frac{5}{3}\right\}\right\}$$

"Solve" can also be used explicitly to change the form of an equation to $y = m\,x + b$ form.

solve 3x + 5y = 11 for y

full Wolfram|Alpha results

```
Solve[3*x + 5*y == 11, y, MaxExtraConditions -> Automatic]
```

$$\left\{\left\{y \to \frac{11}{5} - \frac{3\,x}{5}\right\}\right\}$$

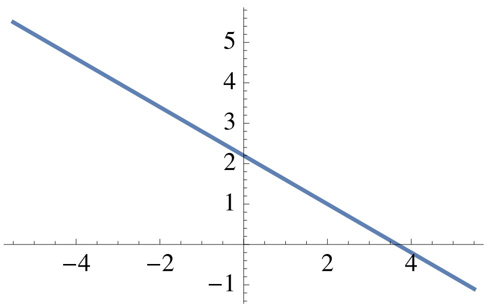
So far in this chapter, the coefficients in the equations have been integers, but Wolfram|Alpha Notebook Edition can work with equations where coefficients are fractions as well.

graph y = −3x/5 + 11/5

related computations

full Wolfram|Alpha results

```
Plot[-3*(x/5) + 11/5, {x, -5.5, 5.5}]
```



Coefficients with decimals work identically as well.

The built-in knowledge in Wolfram|Alpha Notebook Edition can identify the slope, the x intercept and the y intercept in an equation, whether that equation is in slope-intercept form or in an alternate form.

slope of 3x + 5y = 11

alternate interpretations

full Wolfram|Alpha results

Assuming "slope" is referring to geometry | Use "slope of" as referring to derivative computations instead

```
-3/5
```

$$-\frac{3}{5}$$

y–intercept of 3x + 5y = 11

alternate interpretations full Wolfram|Alpha results

Assuming “y–intercept” is referring to geometry | Use “y” as a variable instead

11/5

$$\frac{11}{5}$$

x–intercept of 3x + 5y = 11

alternate interpretations full Wolfram|Alpha results

Assuming “x–intercept” is referring to geometry | Use “x” as a variable instead

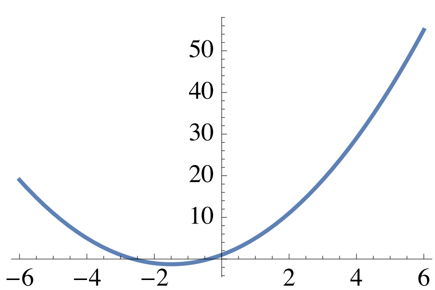
11/3

$$\frac{11}{3}$$

A graph can be created to visualize the curve for a second-degree polynomial by using exactly the same phrase "graph" in the calculation.

graph x^2 + 3x + 1 = 0

```
Plot[x^2 + 3*x + 1 == 0, {x, -6., 6.}]
```

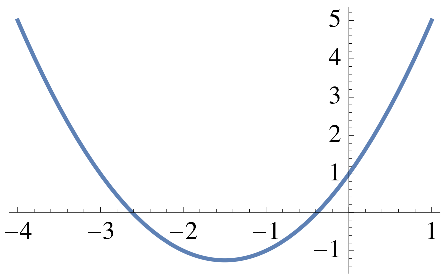


An additional phrase can be added at the end of a calculation to specify lower and upper bounds for the x axis on the graph. In the following calculation, the x axis will be displayed from –4 to 1 to provide a useful view of the points where the curve crosses the x axis.

graph x^2 + 3 x + 1 = 0, x goes from –4 to 1

related computations   full Wolfram|Alpha results

```
Plot[x^2 + 3*x + 1 == 0, {x, -4, 1}]
```



This is one of many cases where a graph can support an understanding of an algebraic concept, and solving the equation for x returns the specific values where the curve crosses the x axis.

solve x^2 + 3x + 1 = 0 for x

step-by-step solution   full Wolfram|Alpha results

```
Solve[x^2 + 3*x + 1 == 0, x]
```

$$\left\{\left\{x \to \frac{1}{2}\left(-3 - \sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2}\left(\sqrt{5} - 3\right)\right\}\right\}$$

The result can be approximated to help with visual comparison with the graph.

result to 5 digits

full Wolfram|Alpha results

```
N[{{x -> (-3 - Sqrt[5])/2}, {x -> (-3 + Sqrt[5])/2}}, 5]
```

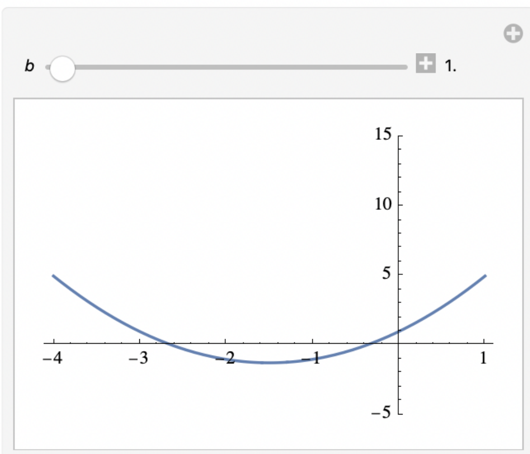
$\{\{x \to -2.6180\}, \{x \to -0.38197\}\}$

When a calculation has more than one result, the results are separated by curly brackets to represent the list of solutions.

Use of the term "varying" also creates a mouse-driven model for other types of equations. The following example varies the symbol b from 1 to 10 to show the effect of varying that parameter on the graph in real time.

graph x^2 + 3 x + b = 0, x goes from −4 to 1, y goes from −5 to 15, vary b from 1 to 10

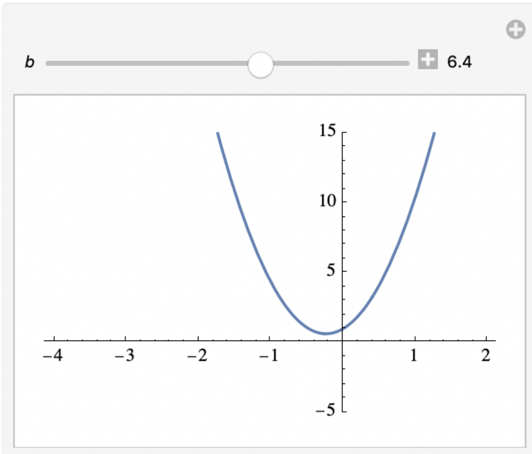
```
Manipulate[Plot[x^2 + 3*x + b == 0, {x, −4, 1}, PlotRange −> {−5, 15}],
  {{b, 11/2}, 1, 10, Appearance −> "Labeled"}]
```



This overall approach can be used to visualize horizontal stretch as well. Rather than varying the third term of the equation, the following example varies the coefficient to the x^2 term. The model shows the resulting graph when this parameter is varied in real time, showing the horizontal stretches and compressions.

graph bx^2 + 3 x + 1 = 0, x goes from −4 to 2, y goes from −5 to 15, vary b from 1 to 10

```
Manipulate[Plot[b*x^2 + 3*x + 1 == 0, {x, -4, 2},
   PlotRange -> {-5, 15}], {{b, 11/2}, 1, 10, Appearance -> "Labeled"}]
```



The text of this book uses 2D typesetting for x^2, while the calculation uses "x^2" with a caret. This mirrors what is currently possible in Wolfram|Alpha Notebook Edition where the **Writing Assistant** palette can be used to create exponents in text for formulas, but calculations do not currently support this level of typesetting.

Working with Exponents

Exponents can be included in any type of calculation in a notebook, and Wolfram|Alpha Notebook Edition includes useful step-by-step solutions for common calculations involving exponents.

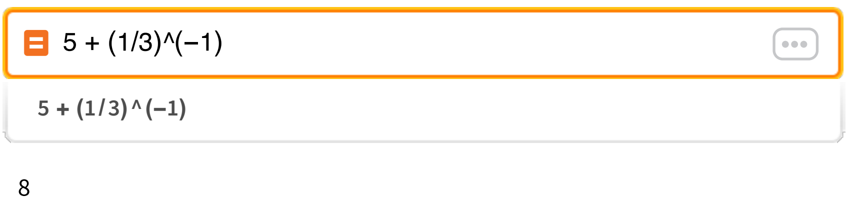
The caret symbol represents an exponent, similar to other software.

5^2 + 3^0

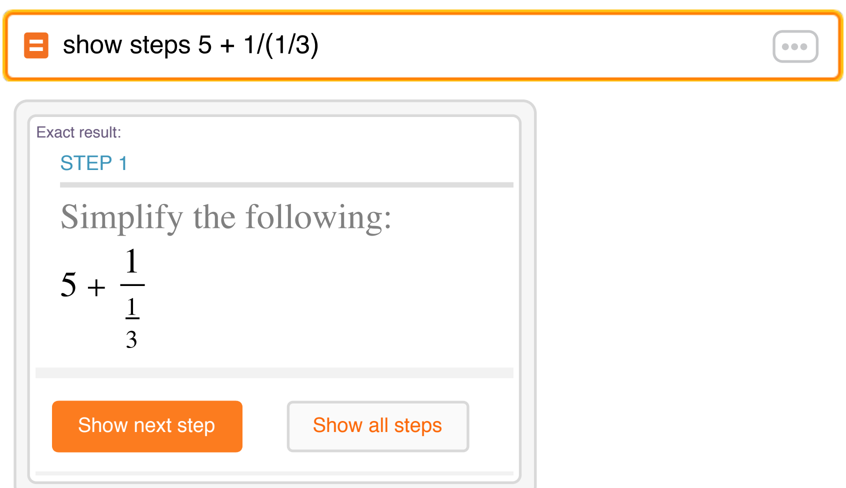
```
5^2 + 3^0
```

The **step-by-step solution** button produces a summary of how to solve arithmetic problems involving exponents.

Negative exponents can be used in calculations with similar notation.



The **step-by-step solution** button confirms the negative exponent represents one over the value, and produces the steps to find the numeric result.



This book often shows only the first step of a step-by-step solution, but the reader should enter the calculations as they read these chapters and work through the step-by-step solutions individually to get a feel for the results for different types of problems.

Exponents in equations can be used when calculating the expanded form.

```
expand (x + 1)^2 + (x − 1)
Expand[(x + 1)^2 + (x − 1)]
```

$x^2 + 3x$

Simplifying the previous result returns an expression with common terms factored out.

```
simplify that
Simplify[3*x + x^2]
```

$x(x + 3)$

The entire process of expanding the expression and factoring out common terms is outlined in more detail in the step-by-step solutions.

```
show steps (x + 1)^2 + (x − 1)
```

Alternate forms

Alternate forms:

STEP 1

Simplify the following:

$(x + 1)^2 + x - 1$

Show next step | Show all steps

A calculation with a natural logarithm can be stated through the use of the common convention of $\ln(x)$.

ln(10) to 100 digits

```
N[Log[10], 100]
```

2.302585092994045684017991454684364207601101488628772976033327900967572609677352480235997205089598298

It is necessary to specify the phrase "to 100 digits" in this calculation; otherwise, the result will be the same as the input for the calculation. The decimal approximations in this book tend to be a modest number of digits just to conserve screen space, but this calculation can easily be extended to one million digits instead of one hundred.

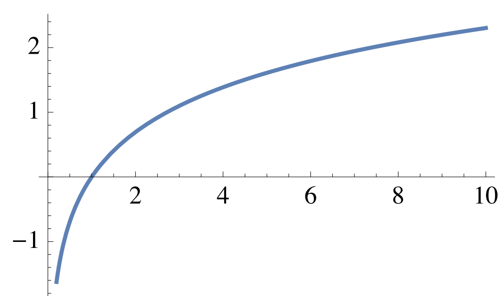
The term "graph" can create a graph based on many formats and types of equations, including an equation with a natural logarithm.

graph y = ln(x), x going from 0 to 10

related computations

full Wolfram|Alpha results

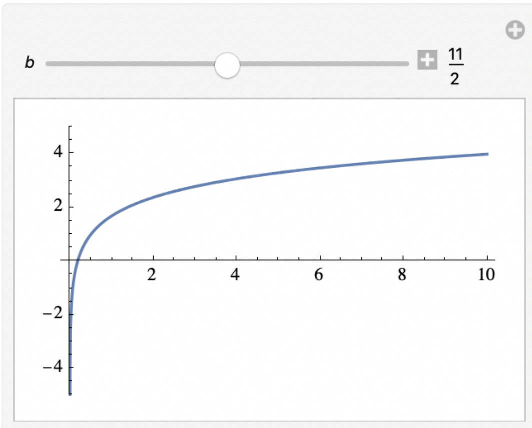
```
Plot[Log[x], {x, 0, 10}]
```



The term "varying" can be used to adjust the value of a coefficient in an equation involving a natural logarithm. In the following calculation, the symbol b is varied from 1 to 10, and the graph is updated in real time based on the new equation.

graph y = ln(b x), x going from 0 to 10, y going from −5 to 5, varying b from 1 to 10

```
Manipulate[Plot[Log[b*x], {x, 0, 10}, PlotRange -> {-5, 5}],
  {{b, 11/2}, 1, 10, Appearance -> "Labeled"}]
```



In addition to moving the slider to adjust the value of the symbol *b*, clicking the plus icon displays a second row of buttons, including an input field to type in new values directly. This can be more precise than moving the slider at times.

Conclusion

Concepts in algebra often involve comparing an algebraic solution to a graph, or working through many specific variations to understand a larger-picture concept. Notebooks are a useful format for a series of related calculations to compare different results, or algebraic solutions against various graphs. Mouse-driven models are also an effective way to visualize variations and focus on a larger-picture concept.

Exercises

1. Calculate the expanded form of $(x-1)(x-2)(x-3)(x-4)(x-5)$.
2. Create a plot of the polynomial in Exercise 1.
3. Create an interactive plot of $\sin(a\,x)$ for x over the interval $[-\pi, \pi]$ that lets you vary the values of a with a slider from 1 to 10.

4. Define the equation of a line with slope 3 and y intercept 5.
5. Evaluate the cube root of $\frac{10\,648}{343}$.
6. Replace the answer from Exercise 5 with its approximate equivalent.
7. Calculate, using "solve," the value of x for which the functions $x + 1$ and $x^2 - 2x + 1$ are equal.
8. Use step-by-step functionality to show how to factor $x^3 - a^6$.
9. Use "line" to find the region in the number line where the inequality $x^2 - 1 > 0$ holds true.
10. Write the step-by-step solutions for Exercise 7.