EXCERPTED FROM

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A NEW KIND OF SCIENCE

SECTION 10.4

Defining Complexity

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Much of what I have done in this book has been concerned in one way or another with phenomena associated with complexity. But just as one does not need a formal definition of life in order to study biology, so also it has not turned out to be necessary so far in this book to have a formal definition of complexity. Nevertheless, following our discussion of randomness in the previous section, we are now in a position to consider how the notion of complexity might be formally defined.

In everyday language, when we say that something seems complex what we typically mean is that we have not managed to find any simple description of it—or at least of those features of it in which we happen to be interested. But the goal of perception and analysis is precisely to find such descriptions, so when we say that something seems complex, what we are effectively saying is that our powers of perception and analysis have failed on it.

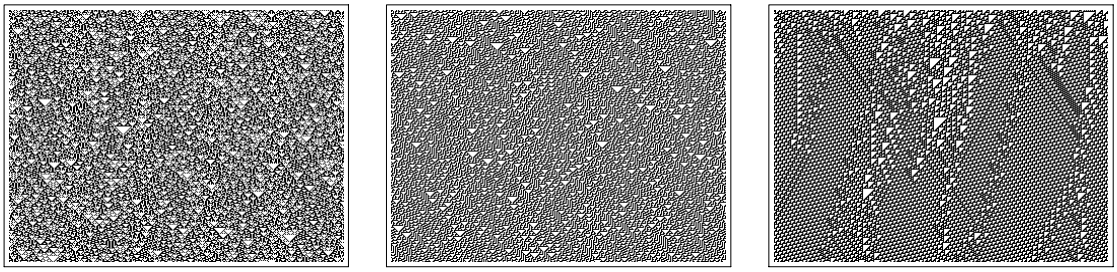
As we discussed two sections ago, there are two ways in which perception and analysis can typically operate. First, they can just throw away details in which we are not interested. And second, they can remove redundancy that is associated with any regularities that they manage to recognize.

The definition of randomness that we discussed in the previous section was based on the failure of the second of these two functions. For what it said was that something should be considered random if our standard methods of perception and analysis could not find any short description from which the thing could faithfully be reproduced.

But in defining complexity we need to consider both functions of perception and analysis. For what we want to know is not whether a simple or short description can be found of every detail of something, but merely whether such a description can be found of those features in which we happen to be interested.

In everyday language, the terms "complexity" and "randomness" are sometimes used almost interchangeably. And for example any of the three pictures at the top of the next page could potentially be referred to as either "quite random" or "quite complex". But if one chooses to look

only at overall features, then typically one would tend to say that the third picture seems more complex than the other two.



Examples of pictures that at an everyday level one might typically describe either as being "quite random" or as being "quite complex".

For even though the detailed placement of black and white cells in the first two pictures does not seem simple to describe, at an overall level these pictures still admit a quite simple description: in essence they just involve a kind of uniform randomness in which every region looks more or less the same as every other. But the third picture shows no such uniformity, even at an overall level. And as a result, we cannot give a short description of it even if we ignore its small-scale details.

Of course, if one goes to an extreme and looks, say, only at how big each picture is, then all three pictures have very short descriptions. And in general how short a description of something one can find will depend on what features of it one wants to capture—which is why one may end up ascribing a different complexity to something when one looks at it for different purposes.

But if one uses a particular method of perception or analysis, then one can always see how short a description this manages to produce. And the shorter the description is, the lower one considers the complexity to be.

But to what extent is it possible to define a notion of complexity that is independent of the details of specific methods of perception and analysis? In this chapter I argue that essentially all common forms of perception and analysis correspond to rather simple programs. And if one is interested in descriptions in which no information is lost—as in the discussion of randomness in the previous section—then as I

mentioned in the previous section, it seems in practice that different simple programs usually agree quite well in their ability or inability to find short descriptions.

But this seems to be considerably less true when one is dealing with descriptions in which information can be lost. For it is rather common to see cases in which only a few features of a system may be difficult to describe—and depending on whether or not a given program happens to be sensitive to these features it can ascribe either a quite high or a quite low complexity to the system.

Nevertheless, as a practical matter, by far the most common way in which we determine levels of complexity is by using our eyes and our powers of visual perception. So in practice what we most often mean when we say that something seems complex is that the particular processes that are involved in human visual perception have failed to extract a short description.

And indeed I suspect that even below the level of conscious thought our brains already have a rather definite notion of complexity. For when we are presented with a complex image, our eyes tend to dwell on it, presumably in an effort to give our brains a chance to extract a simple description.

If we can find no simple features whatsoever—as in the case of perfect randomness—then we tend to lose interest. But somehow the images that draw us in the most—and typically that we find the most aesthetically pleasing—are those for which some features are simple for us to describe, but others have no short description that can be found by any of our standard processes of visual perception.

Before the discoveries in this book, one might have thought that to create anything with a significant level of apparent complexity would necessarily require a procedure which itself had significant complexity. But what we have discovered in this book is that in fact there are remarkably simple programs that produce behavior of great complexity. And what this means—as the images in this book repeatedly demonstrate—is that in the end it is rather easy to make pictures for which our visual system can find no simple overall description.