EXCERPTED FROM

# STEPHEN WOLFRAM

# A NEW KIND OF SCIENCE

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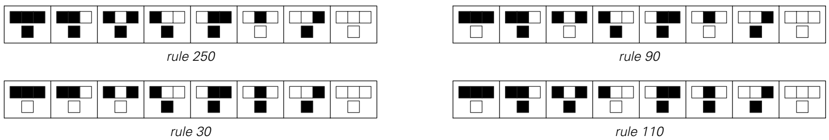
SECTION 3.2

## *More Cellular Automata*

## More Cellular Automata

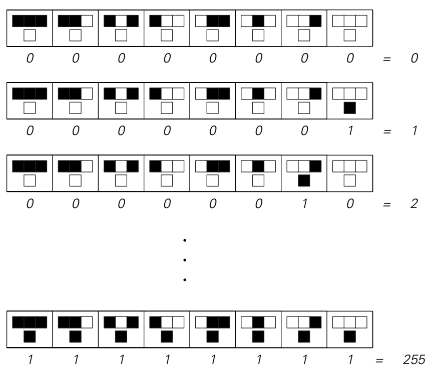
The pictures below show the rules used in the four cellular automata on the facing page. The overall structure of these rules is the same in each case; what differs is the specific choice of new colors for each possible combination of previous colors for a cell and its two neighbors.




The rules used for the four examples of cellular automata on the facing page. In each case, these specify the new color of a cell for each possible combination of colors of that cell and its immediate neighbors on the previous step. The rules are numbered according to the scheme described below.

There turn out to be a total of 256 possible sets of choices that can be made. And following my original work on cellular automata these choices can be numbered from 0 to 255, as in the picture below.

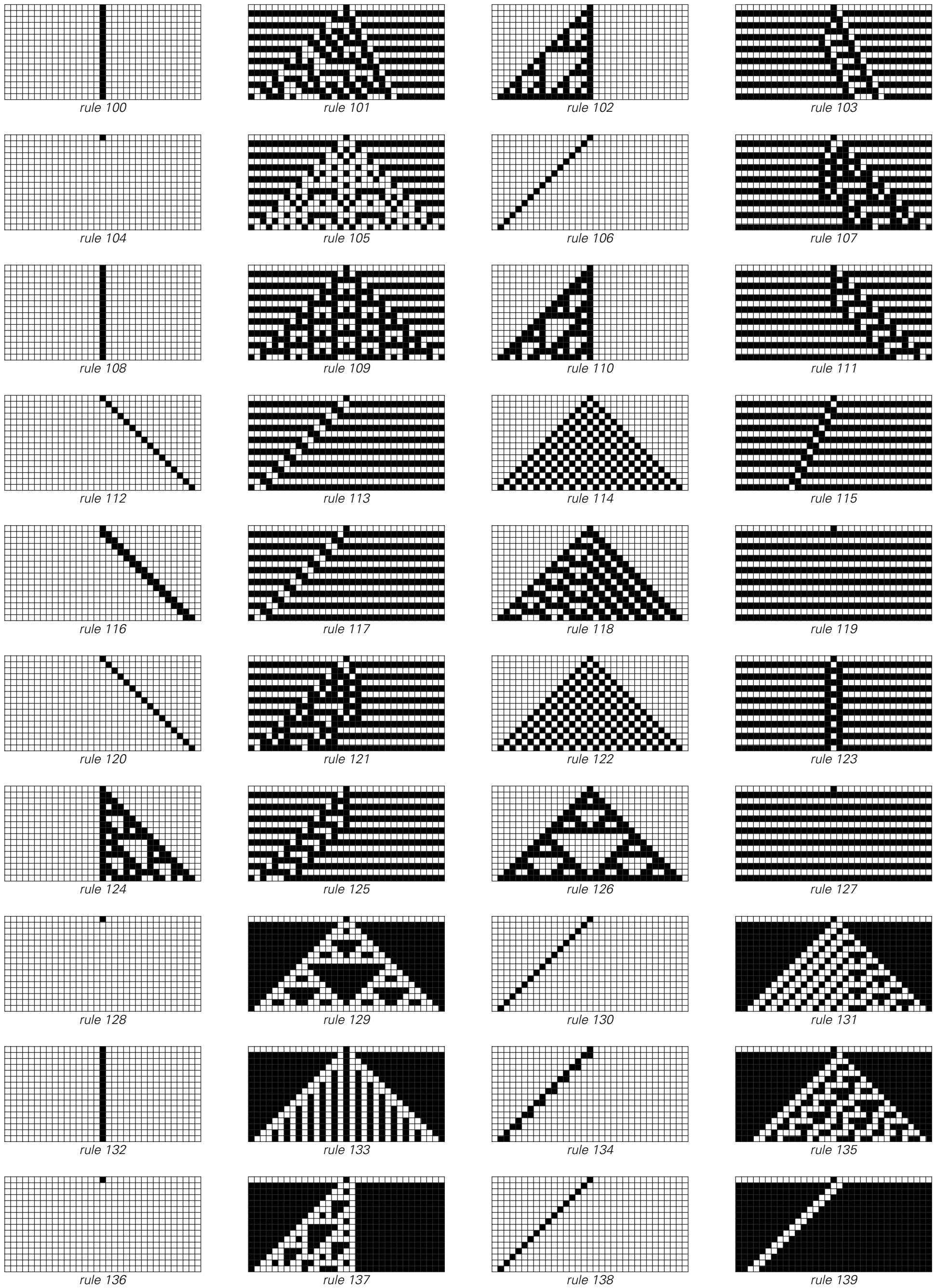
The sequence of 256 possible cellular automaton rules of the kind shown above. As indicated, the rules can conveniently be numbered from 0 to 255. The number assigned is such that when written in base 2, it gives a sequence of 0's and 1's that correspond to the sequence of new colors chosen for each of the eight possible cases covered by the rule.



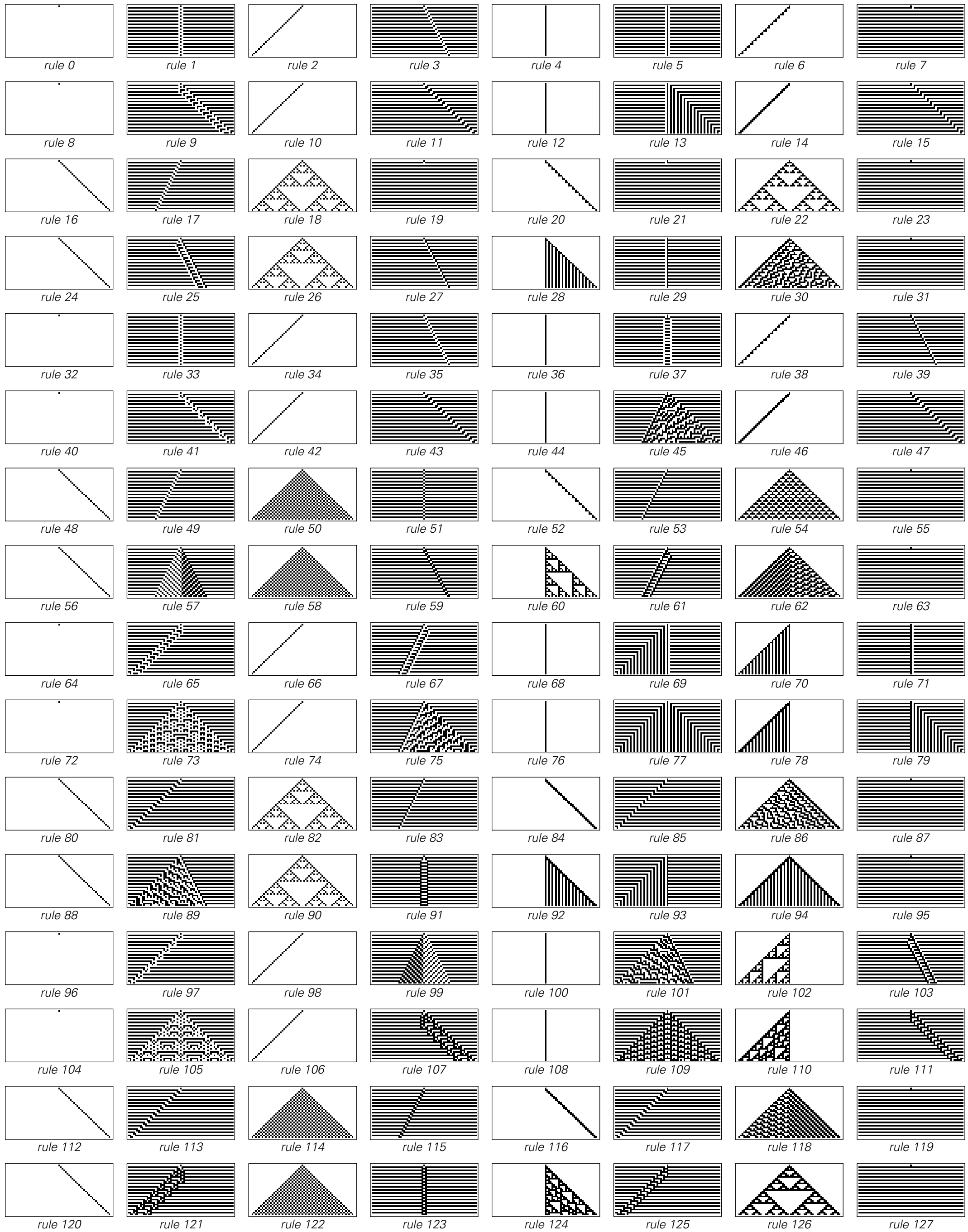

But so how do cellular automata with all these different rules behave? The next page shows a few examples in detail, while the following two pages show what happens in all 256 possible cases.

At first, the diversity of what one sees is a little overwhelming. But on closer investigation, definite themes begin to emerge.

In the very simplest cases, all the cells in the cellular automaton end up just having the same color after one step. Thus, for example, in




Evolution of cellular automata with a sequence of different possible rules, starting in all cases from a single black cell.


rule 0
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rule 14
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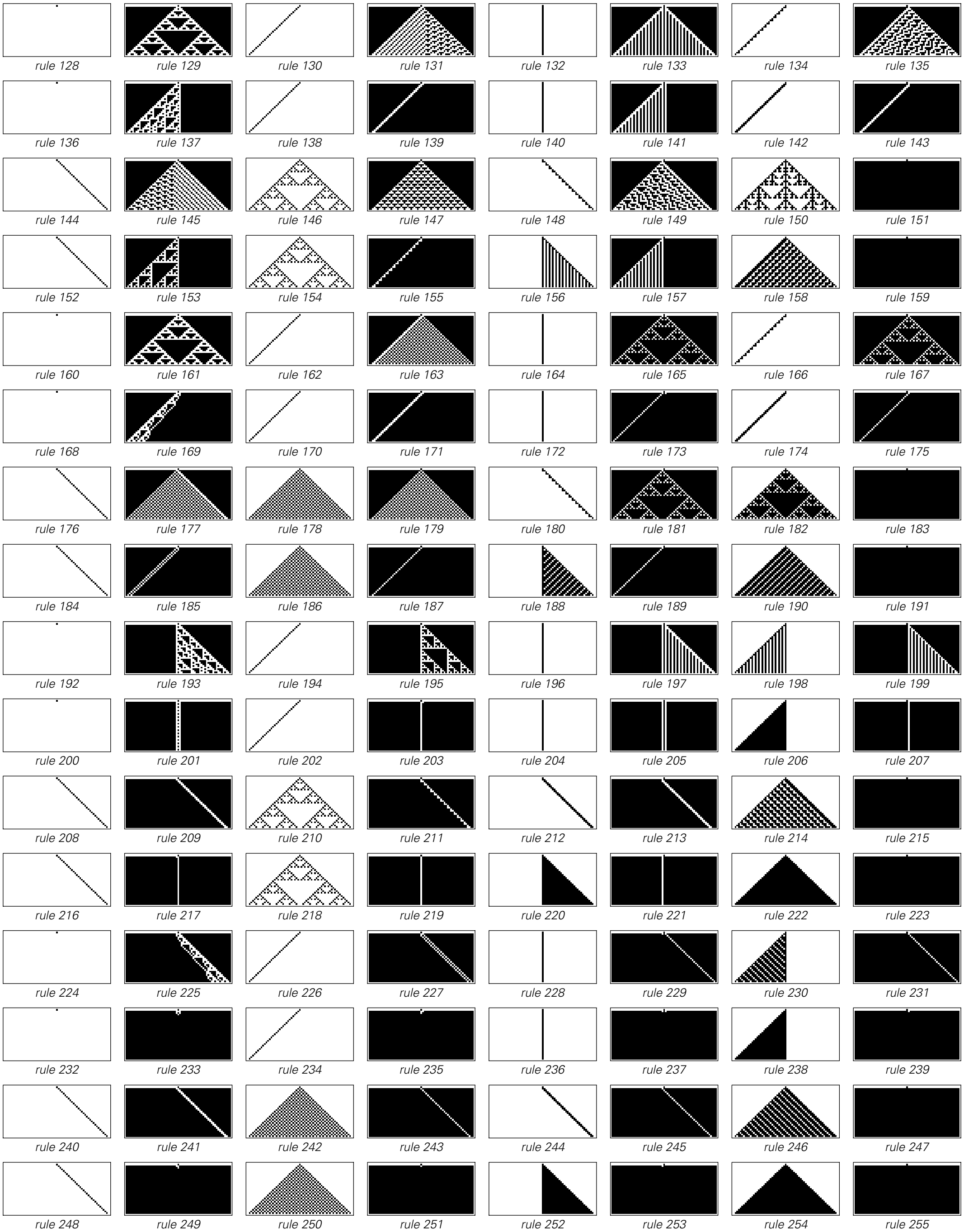

rule 128 rule 129 rule 130 rule 131 rule 132 rule 133 rule 134 rule 135
rule 136 rule 137 rule 138 rule 139 rule 140 rule 141 rule 142 rule 143
rule 144 rule 145 rule 146 rule 147 rule 148 rule 149 rule 150 rule 151
rule 152 rule 153 rule 154 rule 155 rule 156 rule 157 rule 158 rule 159
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rule 240 rule 241 rule 242 rule 243 rule 244 rule 245 rule 246 rule 247
rule 248 rule 249 rule 250 rule 251 rule 252 rule 253 rule 254 rule 255

rules 0 and 128 all the cells become white, while in rule 255 all of them become black. There are also rules such as 7 and 127 in which all cells alternate between black and white on successive steps.

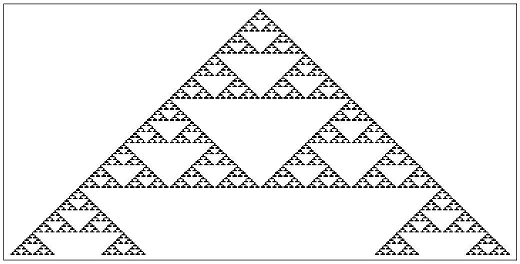
But among the rules shown on the last few pages, the single most common kind of behavior is one in which a pattern consisting of a single cell or a small group of cells persists. Sometimes this pattern remains stationary, as in rules 4 and 123. But in other cases, such as rules 2 and 103, it moves to the left or right.

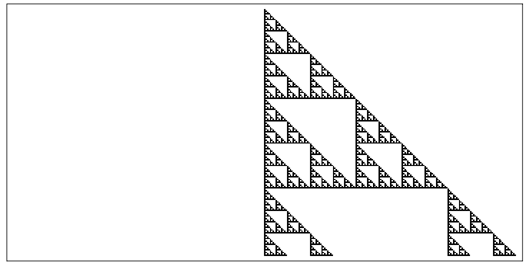
It turns out that the basic structure of the cellular automata discussed here implies that the maximum speed of any such motion must be one cell per step. And in many rules, this maximum speed is achieved—although in rules such as 3 and 103 the average speed is instead only half a cell per step.

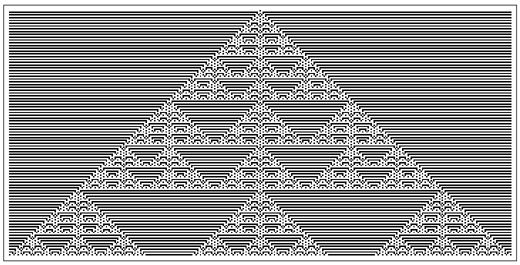
In about two-thirds of all the cellular automata shown on the last few pages, the patterns produced remain of a fixed size. But in about one-third of cases, the patterns instead grow forever. Of such growing patterns, the simplest kind are purely repetitive ones, such as those seen in rules 50 and 109. But while repetitive patterns are by a small margin the most common kind, about 14% of all the cellular automata shown yield more complicated kinds of patterns.

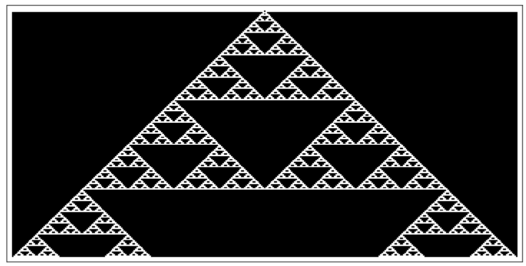
The most common of these are nested patterns, like those on the next page. And it turns out that although 24 rules in all yield such nested patterns, there are only three fundamentally different forms that occur. The simplest and by far the most common is the one exemplified by rules 22 and 60. But as the pictures on the next page show, other nested forms are also possible. (In the case of rule 225, the width of the overall pattern does not grow at a fixed rate, but instead is on average proportional to the square root of the number of steps.)

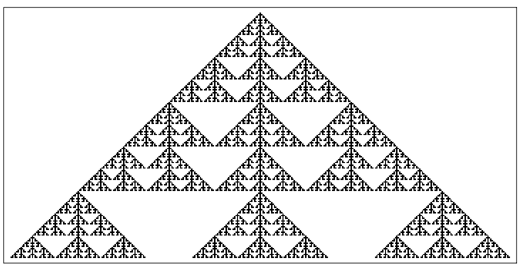
◀ The behavior of all 256 possible cellular automata with rules involving two colors and nearest neighbors. In each case, thirty steps of evolution are shown, starting from a single black cell. Note that some of the rules are related just by interchange of left and right or black and white (e.g. rules 2 and 16 or rules 126 and 129). There are 88 fundamentally inequivalent such elementary rules.

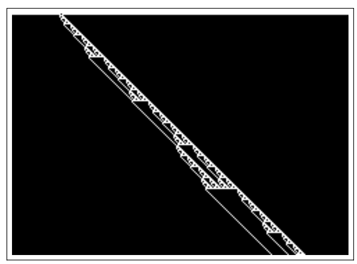

*rule 22*

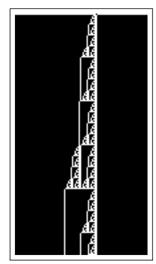

*rule 60*


*rule 105*


*rule 129*


*rule 150*

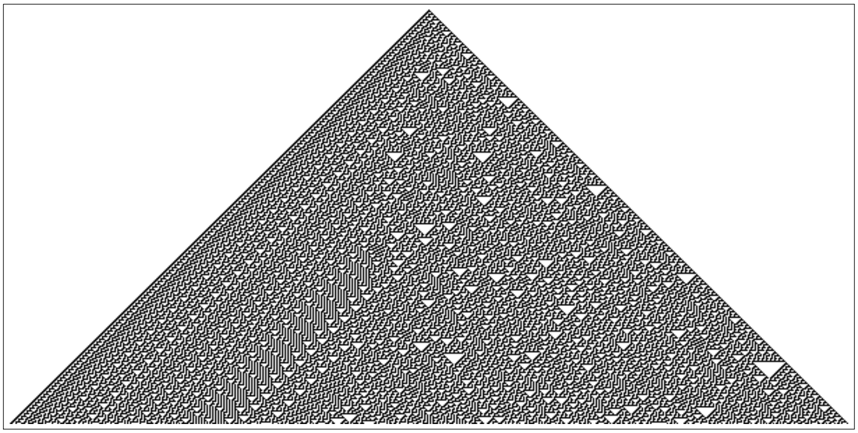

*rule 225*

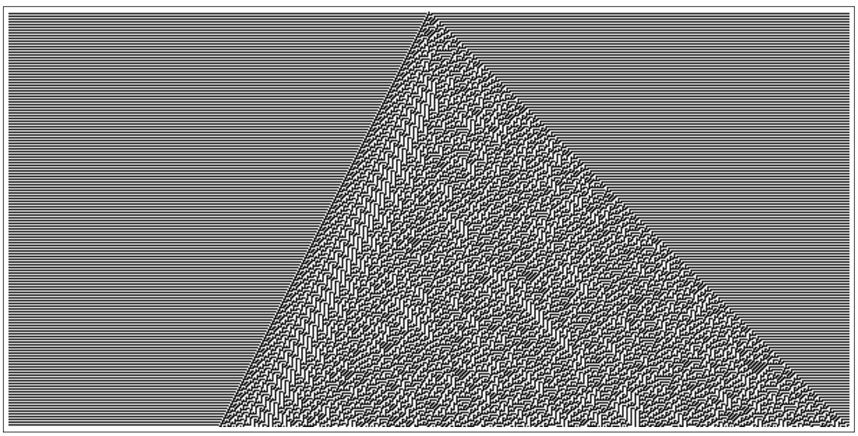

*rule 225 (shifted)*

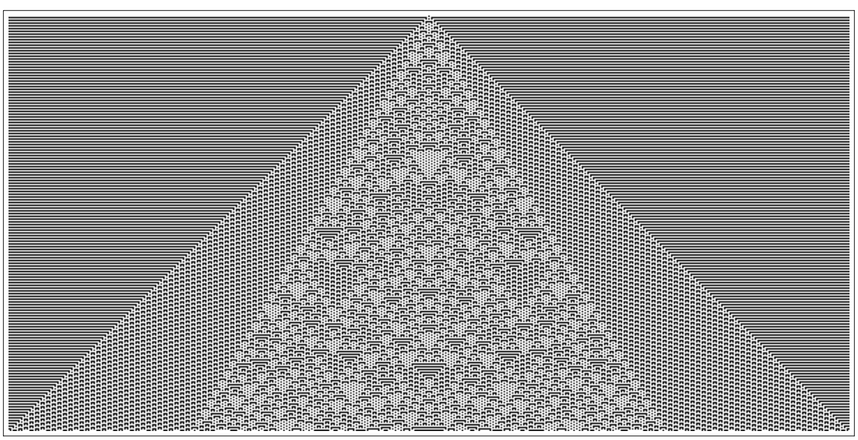
Examples of cellular automata that produce nested or fractal patterns. Rule 22—like rule 90 from page 26—gives a pattern with fractal dimension *Log[2, 3]* $\simeq$ *1.59*; rule 150 gives one with fractal dimension *Log[2, 1 +* $\sqrt{5}$ *]* $\simeq$ *1.69*. The width of the pattern obtained from rule 225 increases like the square root of the number of steps.

Repetition and nesting are widespread themes in many cellular automata. But as we saw in the previous chapter, it is also possible for cellular automata to produce patterns that seem in many respects random. And out of the 256 rules discussed here, it turns out that 10 yield such apparent randomness. There are three basic forms, as illustrated on the facing page.

Examples of cellular automata that produce patterns with many apparently random features. Three hundred steps of evolution are shown, starting in each case from a single black cell. ▶


rule 30


rule 45

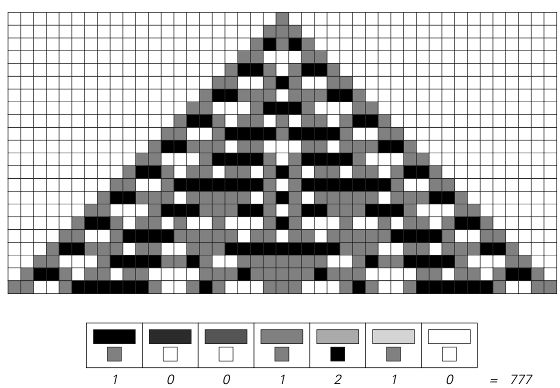

rule 73

Beyond randomness, the last example in the previous chapter was rule 110: a cellular automaton whose behavior becomes partitioned into a complex mixture of regular and irregular parts. This particular cellular automaton is essentially unique among the 256 rules considered here: of the four cases in which such behavior is seen, all are equivalent if one just interchanges the roles of left and right or black and white.

So what about more complicated cellular automaton rules?

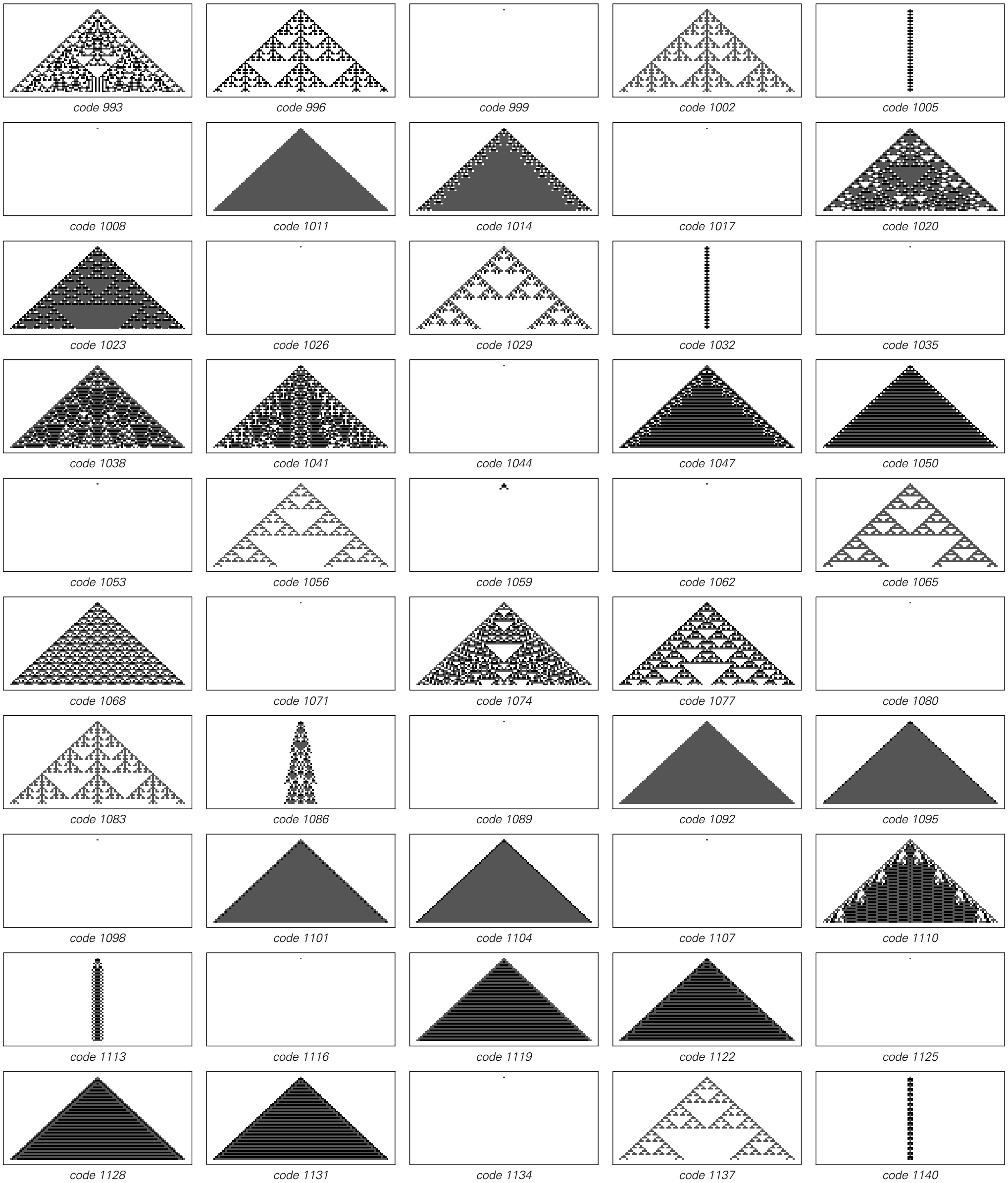
The 256 "elementary" rules that we have discussed so far are by most measures the simplest possible—and were the first ones I studied. But one can for example also look at rules that involve three colors, rather than two, so that cells can not only be black and white, but also gray. The total number of possible rules of this kind turns out to be immense—7,625,597,484,987 in all—but by considering only so-called "totalistic" ones, the number becomes much more manageable.

The idea of a totalistic rule is to take the new color of each cell to depend only on the average color of neighboring cells, and not on their individual colors. The picture below shows one example of how this works. And with three possible colors for each cell, there are 2187 possible totalistic rules, each of which can conveniently be identified by a code number as illustrated in the picture. The facing page shows a representative sequence of such rules.




Example of a totalistic cellular automaton with three possible colors for each cell. The rule is set up so that the new color of every cell is determined by the average of the previous colors of the cell and its immediate neighbors. With 0 representing white, 1 gray and 2 black, the rightmost element of the rule gives the result for average color 0, while the element immediately to its left gives the result for average color 1/3—and so on. Interpreting the sequence of new colors as a sequence of base 3 digits, one can assign a code number to each totalistic rule.

We might have expected that by allowing three colors rather than two we would immediately get noticeably more complicated behavior.

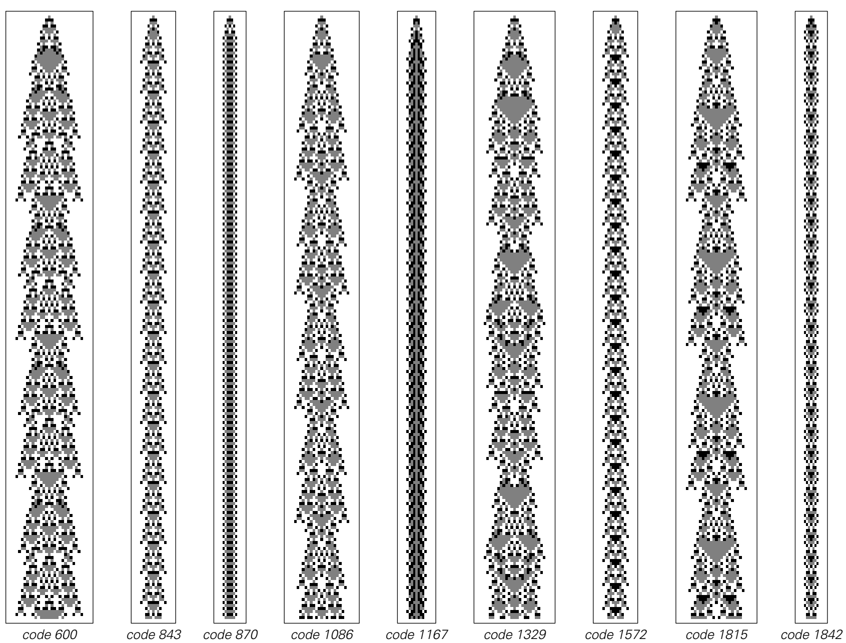



A sequence of totalistic cellular automata with three possible colors for each cell. Although their basic rules are more complicated, the cellular automata shown here do not seem to have fundamentally more complicated behavior than the two-color cellular automata shown on previous pages. Note that in the sequence of rules shown here, those that change the white background are not included. The symmetry of all the patterns is a consequence of the basic structure of totalistic rules.

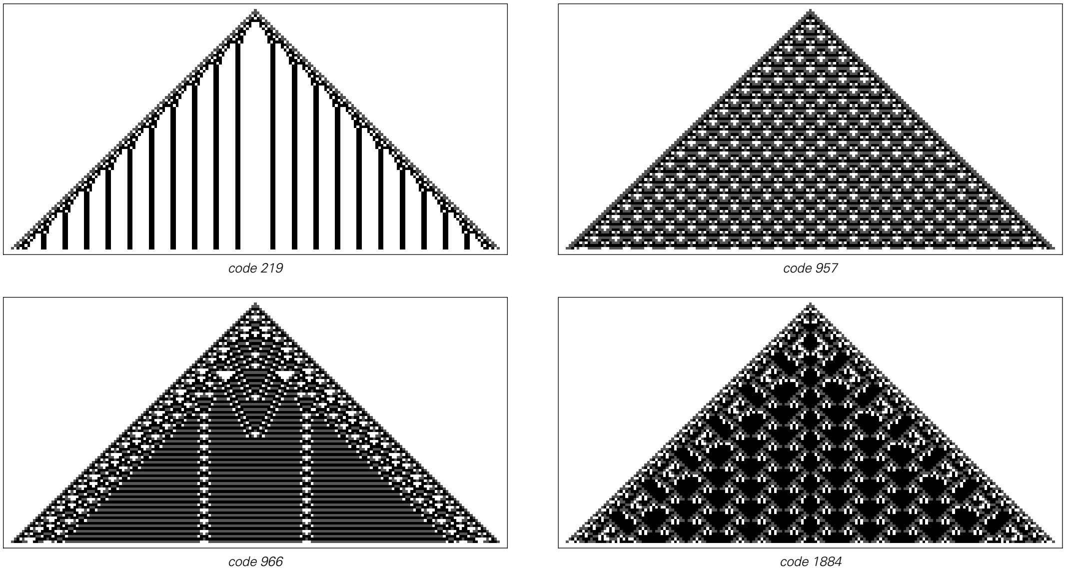
But in fact the behavior we see on the previous page is not unlike what we already saw in many elementary cellular automata a few pages back. Having more complicated underlying rules has not, it seems, led to much greater complexity in overall behavior.

And indeed, this is a first indication of an important general phenomenon: that at least beyond a certain point, adding complexity to the underlying rules for a system does not ultimately lead to more complex overall behavior. And so for example, in the case of cellular automata, it seems that all the essential ingredients needed to produce even the most complex behavior already exist in elementary rules.

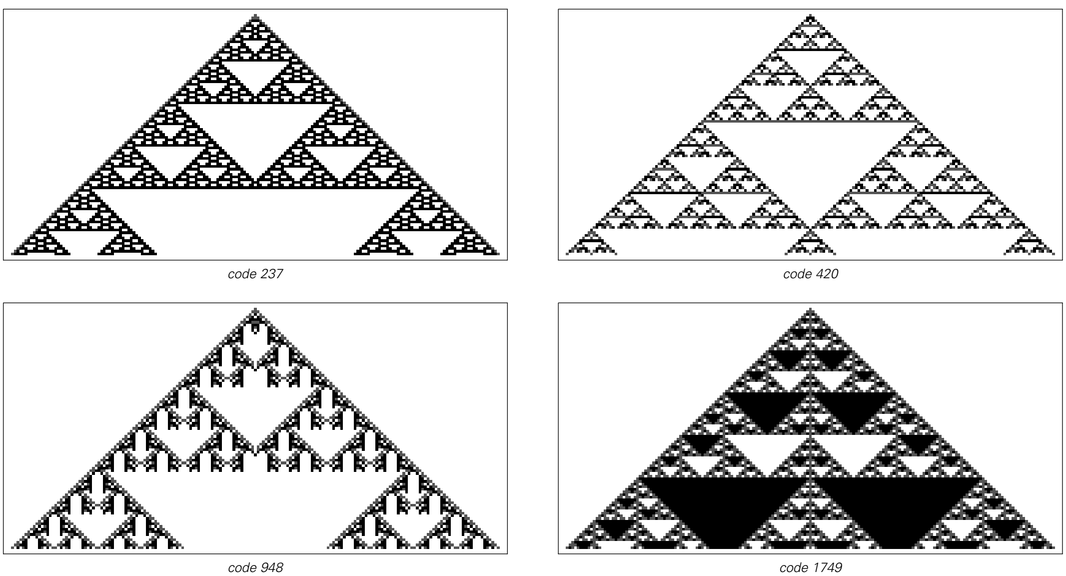
Using more complicated rules may be convenient if one wants, say, to reproduce the details of particular natural systems, but it does not add fundamentally new features. Indeed, looking at the pictures on the previous page one sees exactly the same basic themes as in elementary cellular automata. There are some patterns that attain a definite size, then repeat forever, as shown below, others that continue to grow, but have a repetitive form, as at the top of the facing page, and still others that produce nested or fractal patterns, as at the bottom of the page.



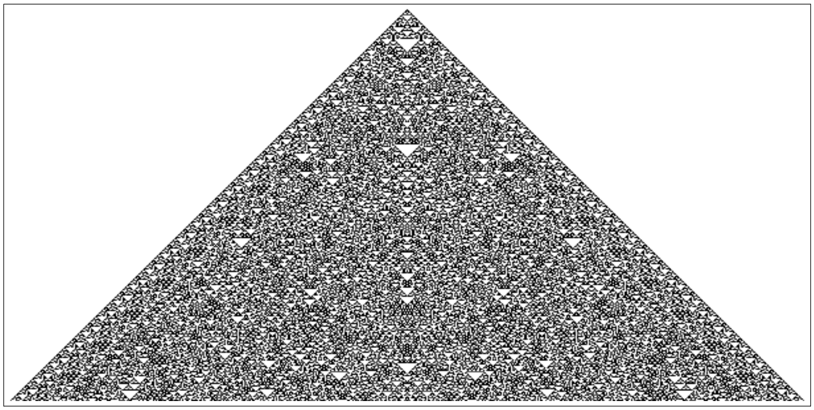

Examples of three-color totalistic rules that yield patterns which attain a certain size, then repeat forever. The maximum repetition period is found to be 78 steps, and is achieved by the rule with code number 1329. In the pictures shown here and on the following pages, the initial condition used contains a single gray cell.



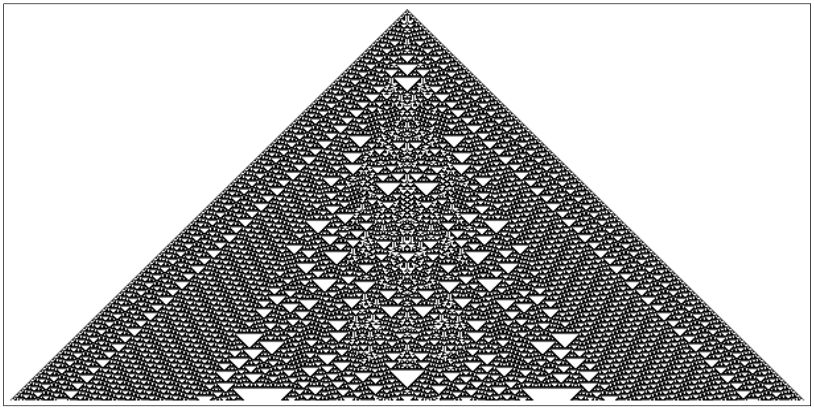

Examples of three-color totalistic rules that yield patterns which grow forever but have a fundamentally repetitive structure.



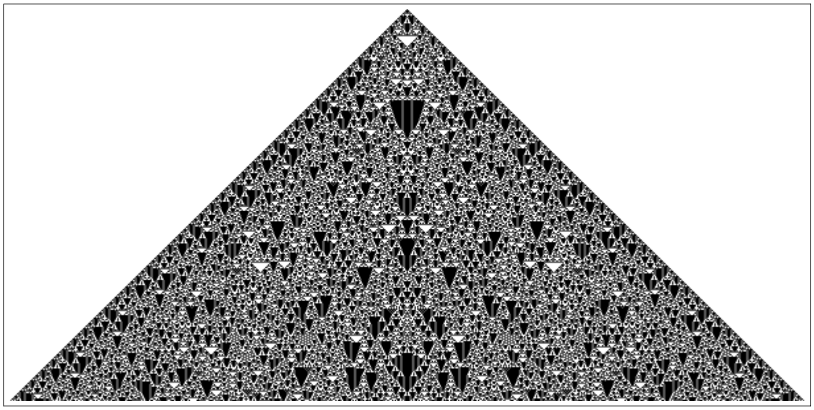

Examples of three-color totalistic rules which yield nested patterns. In most cases, these patterns have an overall form that is similar to what was found with two-color rules. But code 420, for example, yields a pattern with a slightly different structure.



*code 177*



*code 912*



*code 2040*

Examples of three-color totalistic rules that yield patterns with seemingly random features. Three hundred steps of evolution are shown in each case.

In detail, some of the patterns are definitely more complicated than those seen in elementary rules. But at the level of overall behavior, there are no fundamental differences. And in the case of nested patterns even the specific structures seen are usually the same as for elementary rules. Thus, for example, the structure in codes 237 and 948 is the most common, followed by the one in code 1749. The only new structure not already seen in elementary rules is the one in code 420—but this occurs only quite rarely.

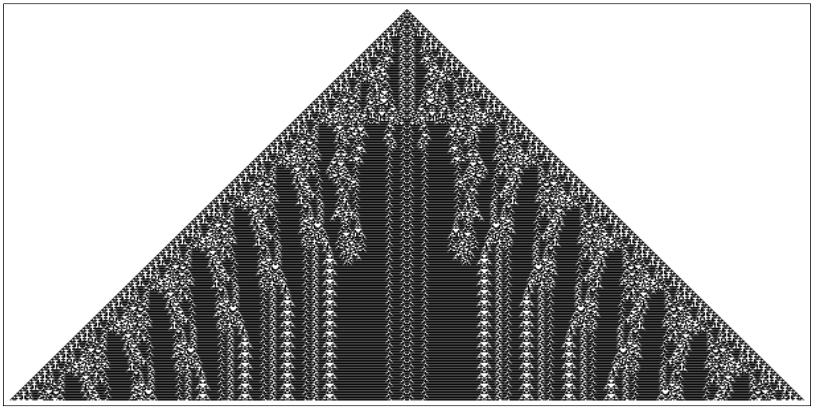
About 85% of all three-color totalistic cellular automata produce behavior that is ultimately quite regular. But just as in elementary cellular automata, there are some rules that yield behavior that seems in many respects random. A few examples of this are given on the facing page.

Beyond fairly uniform random behavior, there are also cases similar to elementary rule 110 in which definite structures are produced that interact in complicated ways. The next page gives a few examples. In the first case shown, the pattern becomes repetitive after about 150 steps. In the other two cases, however, it is much less clear what will ultimately happen. The following pages continue these patterns for 3000 steps. But even after this many steps it is still quite unclear what the final behavior will be.

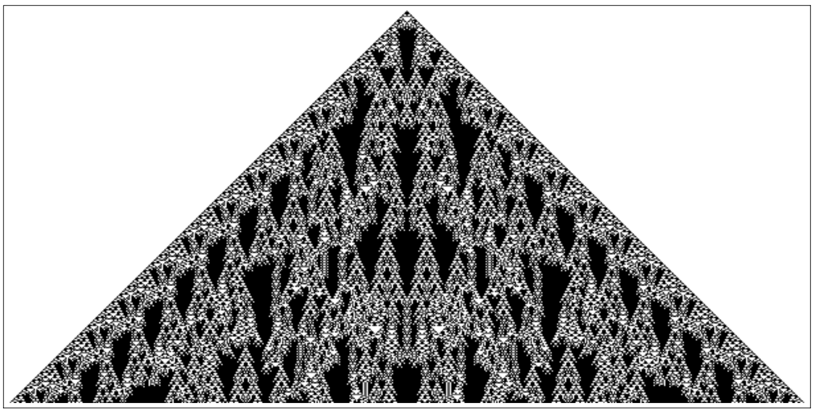
Looking at pictures like these, it is at first difficult to believe that they can be generated just by following very simple underlying cellular automaton rules. And indeed, even if one accepts this, there is still a tendency to assume that somehow what one sees must be a consequence of some very special feature of cellular automata.

As it turns out, complexity is particularly widespread in cellular automata, and for this reason it is fortunate that cellular automata were the very first systems that I originally decided to study.

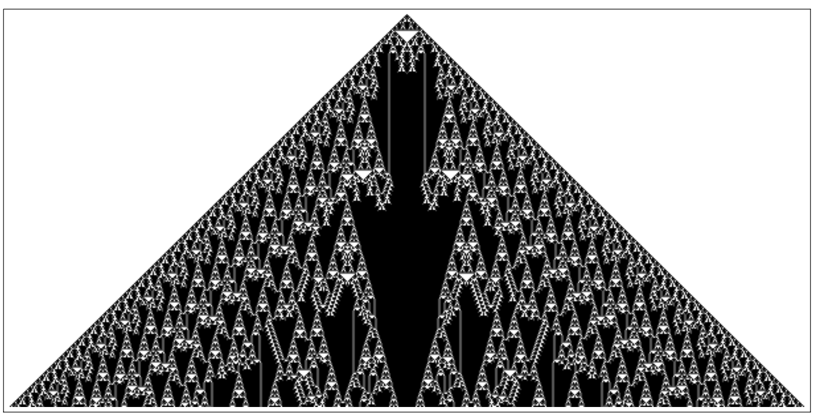
But as we will see in the remainder of this chapter, the fundamental phenomena that we discovered in the previous chapter are in no way restricted to cellular automata. And although cellular automata remain some of the very best examples, we will see that a vast range of utterly different systems all in the end turn out to exhibit extremely similar types of behavior.



code 1041

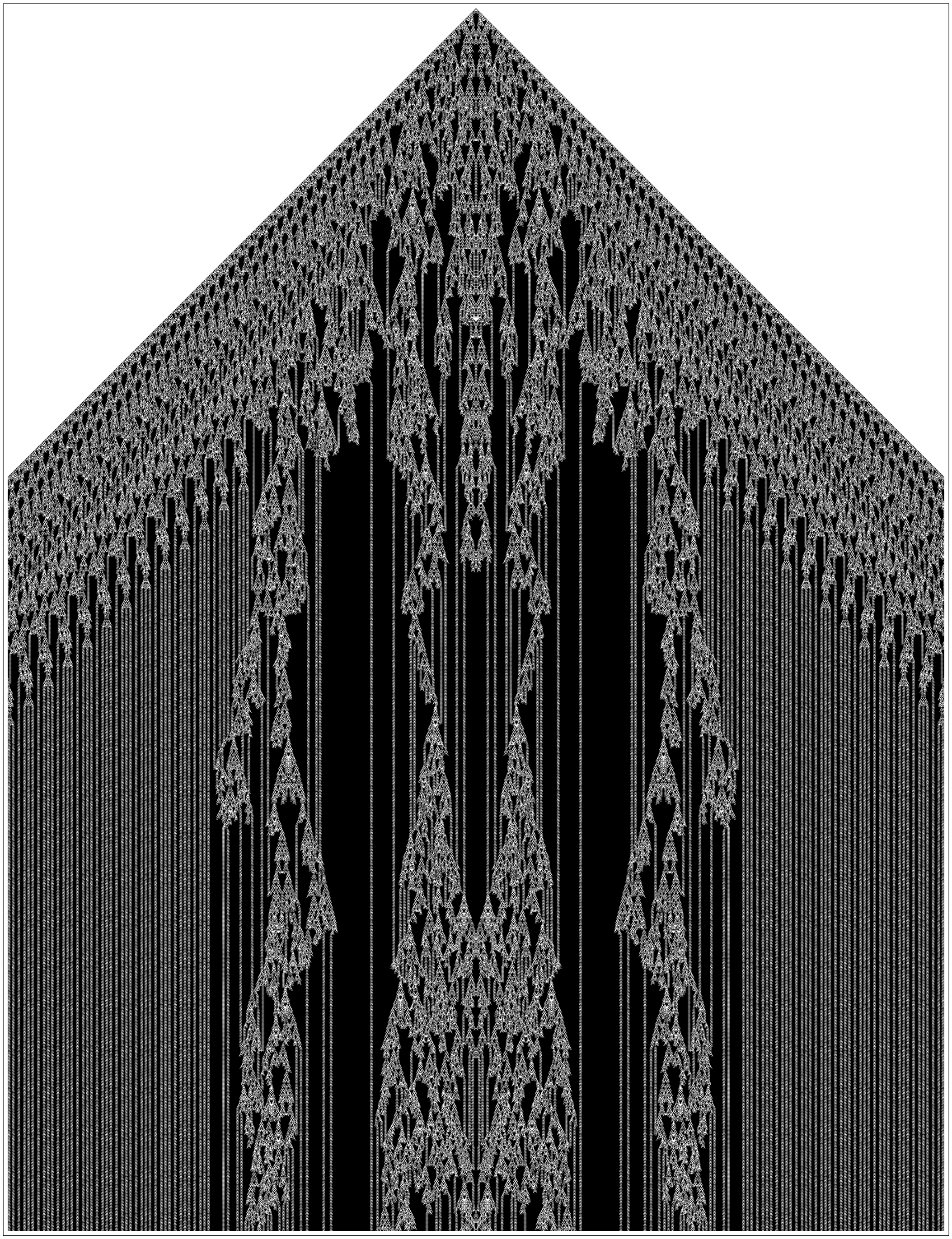


code 1635

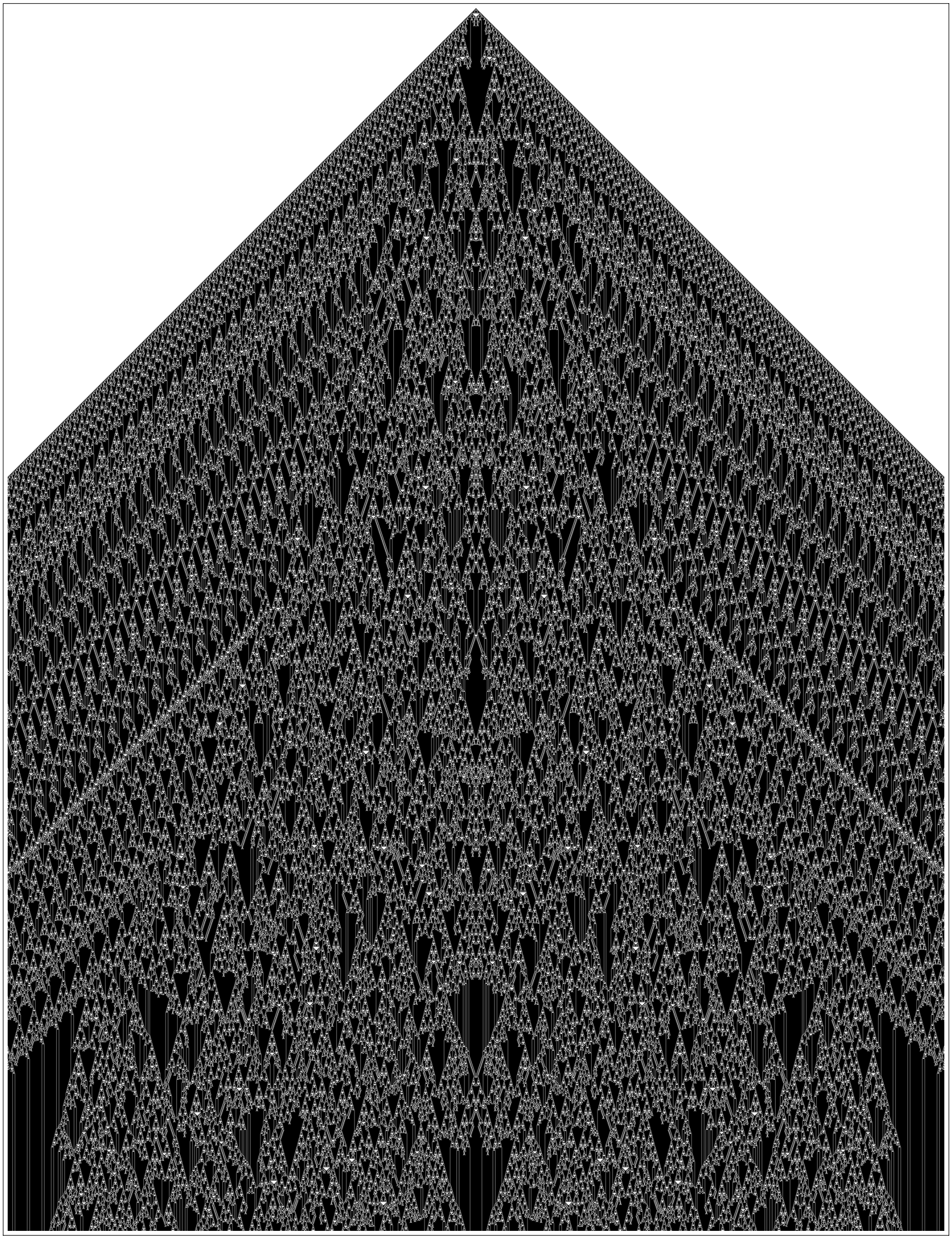


code 2049

Examples of three-color totalistic rules with highly complex behavior showing a mixture of regularity and irregularity. The partitioning into identifiable structures is similar to what we saw in rule 110 on page 32.

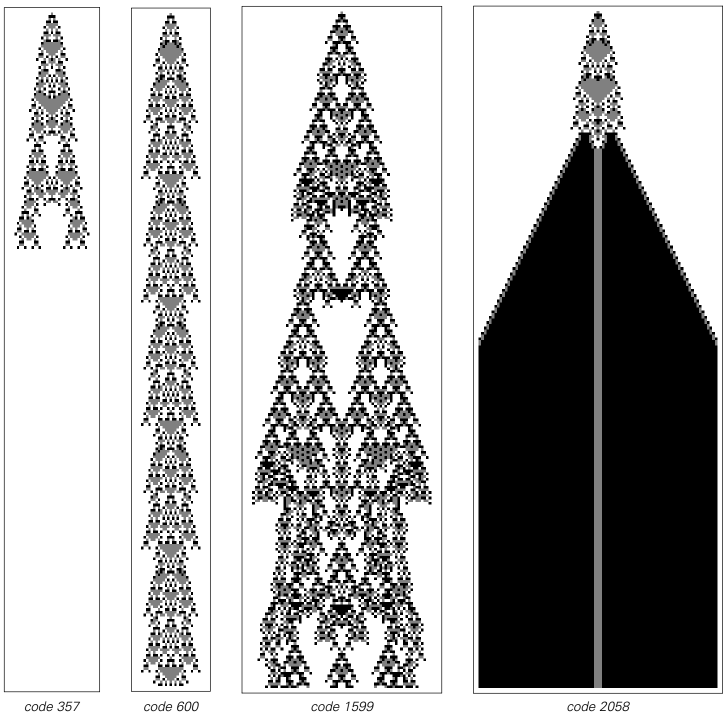


*code 1635*



*code 2049*

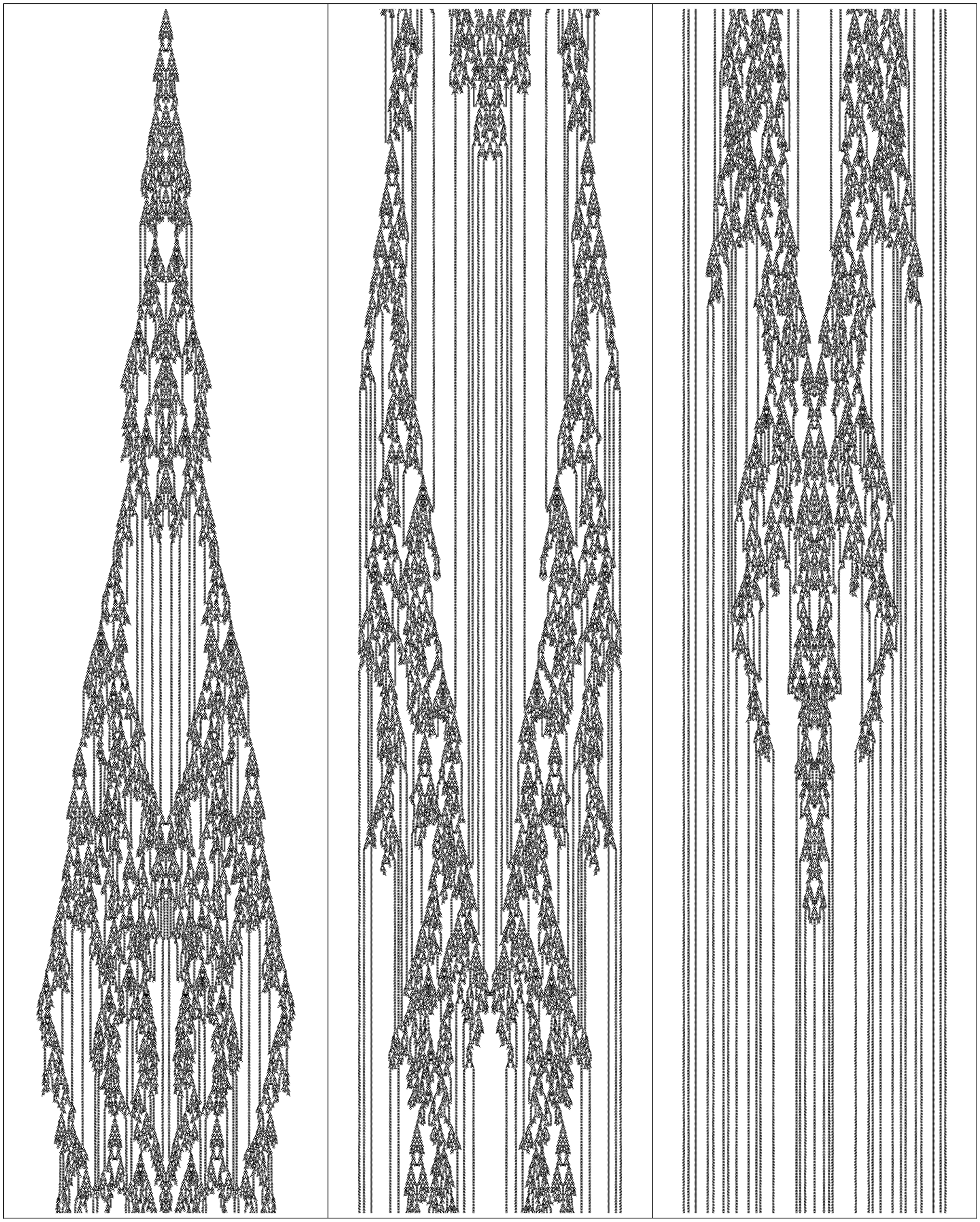
The pictures below show totalistic cellular automata whose overall patterns of growth seem, at least at first, quite complicated. But it turns out that after only about 100 steps, three out of four of these patterns have resolved into simple forms.




Examples of rules that yield patterns which seem to be on the edge between growth and extinction. For all but code 1599, the fate of these patterns in fact becomes clear after less than 100 steps. A total of 250 steps are shown here.

The one remaining pattern is, however, much more complicated. As shown on the next page, for several thousand steps it simply grows, albeit somewhat irregularly. But then its growth becomes slower. And inside the pattern parts begin to die out. Yet there continue to be occasional bursts of growth. But finally, after a total of 8282 steps, the pattern resolves into 31 simple repetitive structures.

◀ Three thousand steps in the evolution of the last two cellular automata from page 66. Despite the simplicity of their underlying rules, the final patterns produced show immense complexity. In neither case is it clear what the final outcome will be—whether apparent randomness will take over, or whether a simple repetitive form will emerge.



*code 1599*

Nine thousand steps in the evolution of the three-color totalistic cellular automaton with code number 1599. Starting from a single gray cell, each column corresponds to 3000 steps. The outcome of the evolution finally becomes clear after 8282 steps, when the pattern resolves into 31 simple repetitive structures.