EXCERPTED FROM

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A NEW KIND OF SCIENCE

SECTION 4.10

Continuous Versus Discrete Systems

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One of the most obvious differences between my approach to science based on simple programs and the traditional approach based on mathematical equations is that programs tend to involve discrete elements while equations tend to involve continuous quantities.

But how significant is this difference in the end?

One might have thought that perhaps the basic phenomenon of complexity that I have identified could only occur in discrete systems. But from the results of the last few sections, we know that this is not the case.

What is true, however, is that the phenomenon was immensely easier to discover in discrete systems than it would have been in continuous ones. Probably complexity is not in any fundamental sense rarer in continuous systems than in discrete ones. But the point is that discrete systems can typically be investigated in a much more direct way than continuous ones.

Indeed, given the rules for a discrete system, it is usually a rather straightforward matter to do a computer experiment to find out how the system will behave. But given an equation for a continuous system, it often requires considerable analysis to work out even approximately how the system will behave. And in fact, in the end one typically has rather little idea which aspects of what one sees are actually genuine features of the system, and which are just artifacts of the particular methods and approximations that one is using to study it.

With all the work that was done on continuous systems in the history of traditional science and mathematics, there were undoubtedly many cases in which effects related to the phenomenon of complexity were seen. But because the basic phenomenon of complexity was not known and was not expected, such effects were probably always dismissed as somehow not being genuine features of the systems being studied. Yet when I came to investigate discrete systems there was no

◀ Solutions to the same equations as on the previous page over a longer period of time. Note the appearance of discrete structures. Particularly in the last picture some details are sensitive to the numerical approximation scheme used in computing the solution to the equation.

possibility of dismissing what I saw in such a way. And as a result I was in a sense forced into recognizing the basic phenomenon of complexity.

But now, armed with the knowledge that this phenomenon exists, it is possible to go back and look again at continuous systems.

And although there are significant technical difficulties, one finds as the last few sections have shown that the phenomenon of complexity can occur in continuous systems just as it does in discrete ones.

It remains much easier to be sure of what is going on in a discrete system than in a continuous one. But I suspect that essentially all of the various phenomena that we have observed in discrete systems in the past several chapters can in fact also be found even in continuous systems with fairly simple rules.