EXCERPTED FROM

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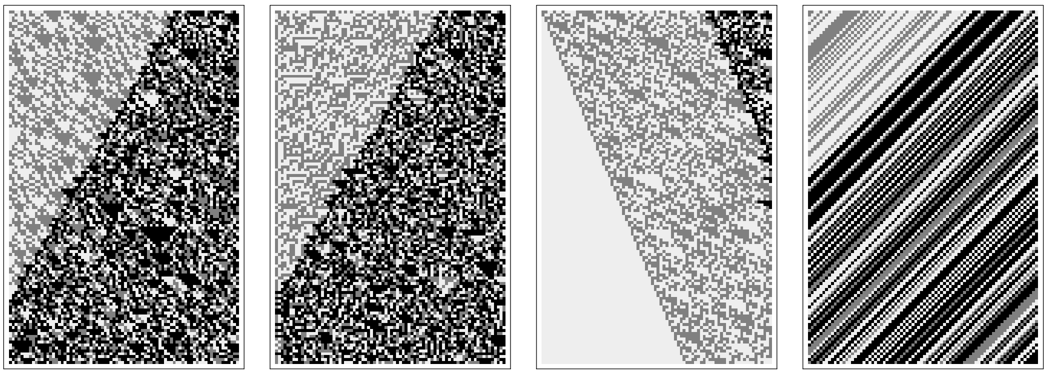
A NEW KIND OF SCIENCE

SECTION 4.8

Continuous Cellular Automata

do not also show sensitive dependence on initial conditions. And indeed the pictures below illustrate that even in such cases changes in digit sequences are progressively amplified—just like in the shift map case (d).



(a) (b) (c) (d)

Differences in digit sequences produced by a small change in initial conditions for the four iterated maps discussed in this section. Cases (a), (b) and (d) exhibit sensitive dependence on initial conditions, in the sense that a change in insignificant digits far to the right eventually grows to affect all digits. Case (c) does not show such sensitivity to initial conditions, but instead always evolves to 0, independent of its initial conditions.

But the crucial point that I will discuss more in Chapter 7 is that the presence of sensitive dependence on initial conditions in systems like (a) and (b) in no way implies that it is what is responsible for the randomness and complexity we see in these systems. And indeed, what looking at the shift map in terms of digit sequences shows us is that this phenomenon on its own can make no contribution at all to what we can reasonably consider the ultimate production of randomness.

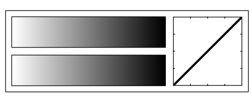
Continuous Cellular Automata

Despite all their differences, the various kinds of programs discussed in the previous chapter have one thing in common: they are all based on elements that can take on only a discrete set of possible forms, typically just colors black and white. And in this chapter, we have introduced a similar kind of discreteness into our study of systems based on numbers by considering digit sequences in which each digit can again have only a discrete set of possible values, typically just 0 and 1.

So now a question that arises is whether all the complexity we have seen in the past three chapters somehow depends on the discreteness of the elements in the systems we have looked at.

And to address this question, what I will do in this section is to consider a generalization of cellular automata in which each cell is not just black or white, but instead can have any of a continuous range of possible levels of gray. One can update the gray level of each cell by using rules that are in a sense a cross between the totalistic cellular automaton rules that we discussed at the beginning of the last chapter and the iterated maps that we just discussed in the previous section.

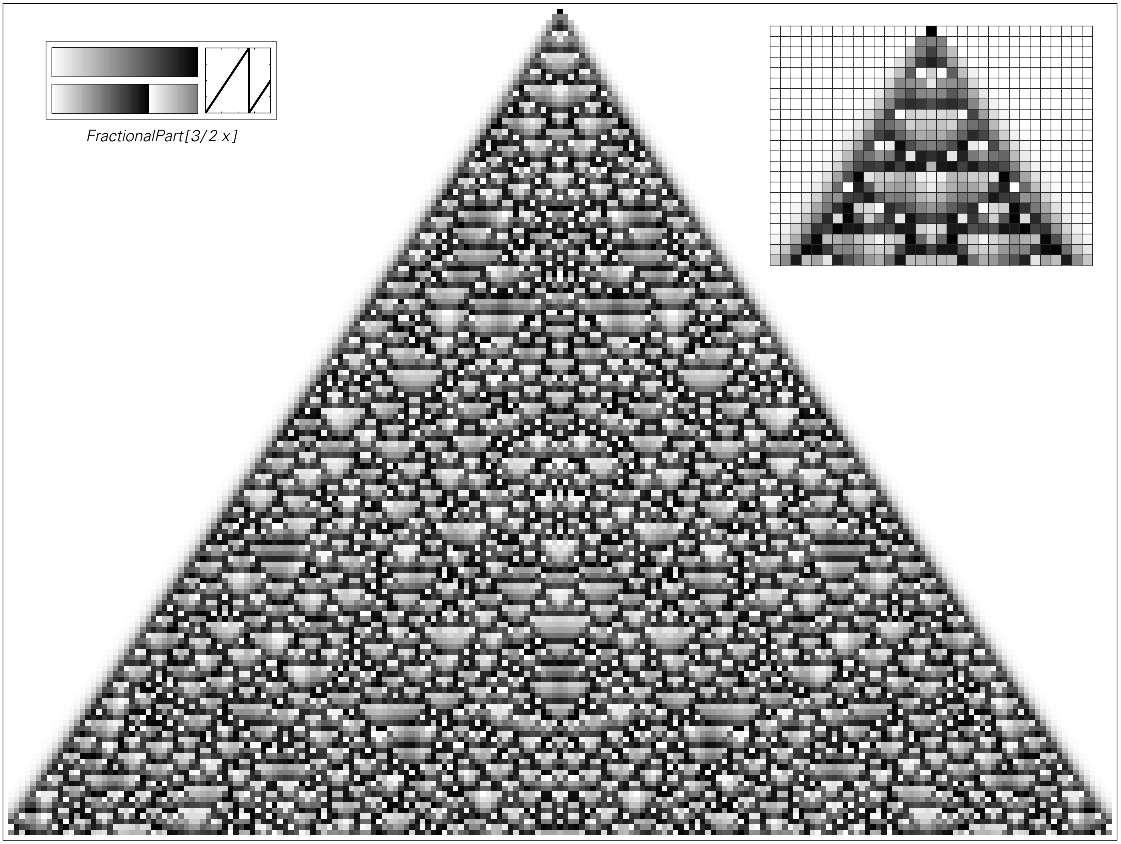
The idea is to look at the average gray level of a cell and its immediate neighbors, and then to get the gray level for that cell at the next step by applying a fixed mapping to the result. The picture below shows a very simple case in which the new gray level of each cell is exactly the average of the one for that cell and its immediate neighbors. Starting from a single black cell, what happens in this case is that the gray essentially just diffuses away, leaving in the end a uniform pattern.



A continuous cellular automaton in which each cell can have any level of gray between white (0) and black (1). The rule shown here takes the new gray level of each cell to be the average of its own gray level and those of its immediate neighbors.

0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0.333	0.333	0.333	0	0	0	0
0	0	0	0.111	0.222	0.333	0.222	0.111	0	0	0
0	0	0.037	0.111	0.222	0.259	0.222	0.111	0.037	0	0
0	0.012	0.049	0.123	0.198	0.235	0.198	0.123	0.049	0.012	0
0.004	0.021	0.062	0.123	0.185	0.21	0.185	0.123	0.062	0.021	0.004

The picture on the facing page shows what happens with a slightly more complicated rule in which the average gray level is multiplied by 3/2, and then only the fractional part is kept if the result of this is greater than 1.



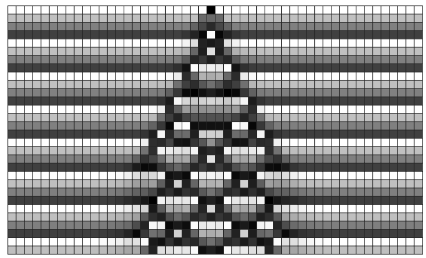
0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0.5	0.5	0.5	0	0	0	0
0	0	0	0.25	0.5	0.75	0.5	0.25	0	0	0
0	0	0.125	0.375	0.75	0.875	0.75	0.375	0.125	0	0
0	0.063	0.25	0.625	0	0.188	0	0.625	0.25	0.063	0
0.031	0.156	0.469	0.438	0.406	0.094	0.406	0.438	0.469	0.156	0.031

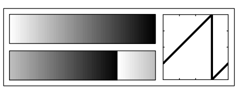
A continuous cellular automaton with a slightly more complicated rule. The rule takes the new gray level of each cell to be the fractional part of the average gray level of the cell and its neighbors multiplied by 3/2. The picture shows that starting from a single black cell, this rule yields behavior of considerable complexity. Note that the operation performed on individual average gray levels is exactly iterated map (a) from page 150.

And what we see is that despite the presence of continuous gray levels, the behavior that is produced exhibits the same kind of complexity that we have seen in many ordinary cellular automata and other systems with discrete underlying elements.

In fact, it turns out that in continuous cellular automata it takes only extremely simple rules to generate behavior of considerable complexity. So as an example the picture below shows a rule that determines the new gray level for a cell by just adding the constant 1/4 to the average gray level for the cell and its immediate neighbors, and then taking the fractional part of the result.

0	0	0	0	0	1	0	0	0	0	0
0.25	0.25	0.25	0.25	0.583	0.583	0.583	0.25	0.25	0.25	0.25
0.5	0.5	0.5	0.611	0.722	0.833	0.722	0.611	0.5	0.5	0.5
0.75	0.75	0.787	0.861	0.972	0.009	0.972	0.861	0.787	0.75	0.75
0	0.012	0.049	0.123	0.864	0.901	0.864	0.123	0.049	0.012	0
0.254	0.271	0.312	0.596	0.88	0.127	0.88	0.596	0.312	0.271	0.254





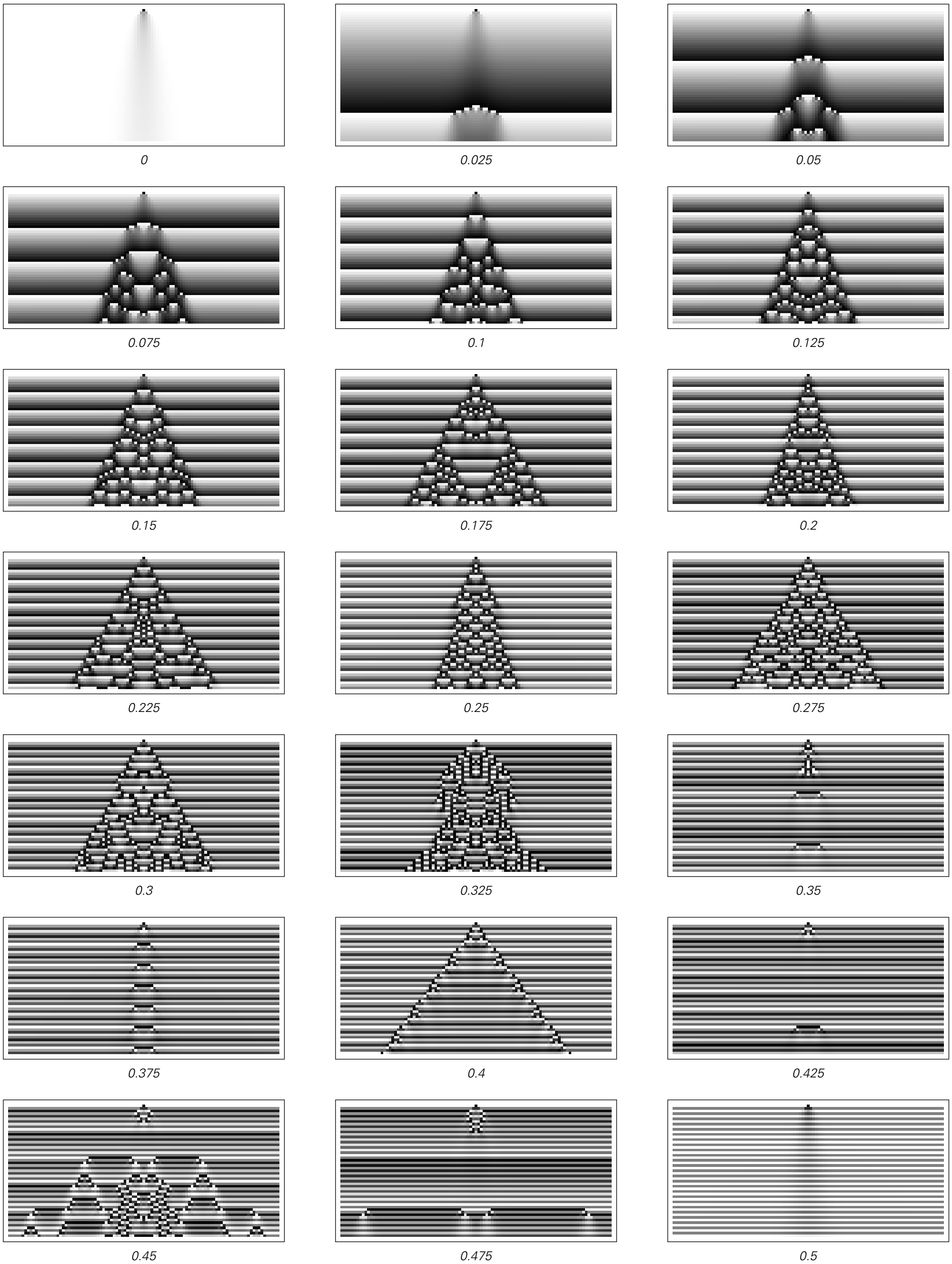
FractionalPart[x+1/4]

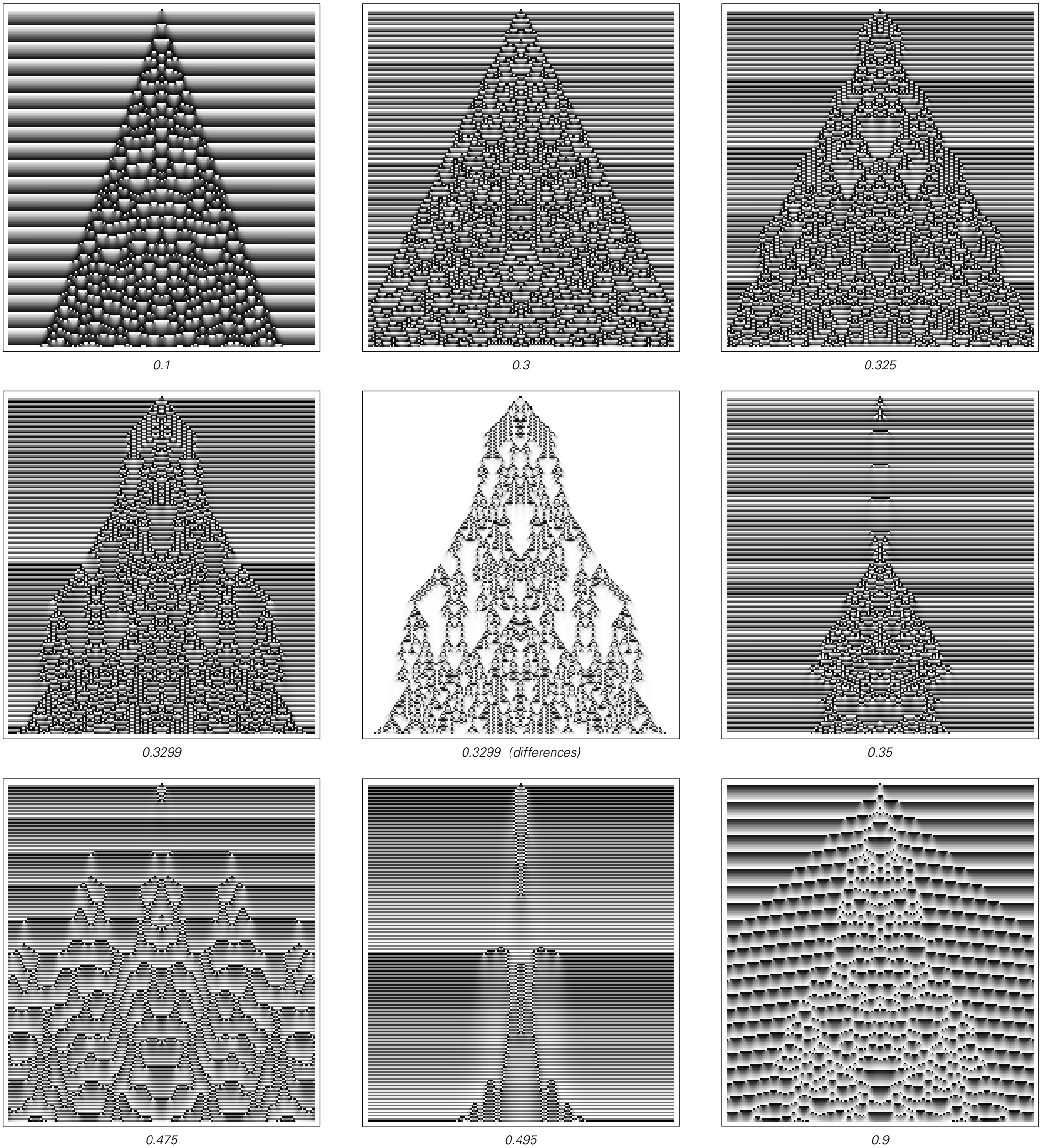
A continuous cellular automaton whose rule adds the constant 1/4 to the average gray level for a cell and its immediate neighbors, and takes the fractional part of the result. The background simply repeats every 4 steps, but the main pattern has a complex and in many respects random form.

The facing page and the one after show what happens when one chooses different values for the constant that is added. A remarkable diversity of behavior is seen. Sometimes the behavior is purely repetitive, but often it has features that seem effectively random.

And in fact, as the picture in the middle of page 160 shows, it is even possible to find cases that exhibit localized structures very much like those occasionally seen in ordinary cellular automata.

Continuous cellular automata with the same kind of rules as in the picture above, but with a variety of different constants being added. Note that it is not so much the size of the constant as properties like its digit sequence that seem to determine the overall form of behavior produced in each case. ▶


0
0.025
0.05
0.075
0.1
0.125
0.15
0.175
0.2
0.225
0.25
0.275
0.3
0.325
0.35
0.375
0.4
0.425
0.45
0.475
0.5



More steps in the evolution of continuous cellular automata with the same kind of rules as on the previous page. In order to remove the uniform stripes, the picture in the middle shows the difference between the gray level of each cell and its immediate neighbor. Note the presence of discrete localized structures even though the underlying rules for the system involve continuous gray levels.