EXCERPTED FROM

STEPHEN WOLFRAM
A NEW KIND OF SCIENCE

SECTION 5.2

Cellular Automata

Traditional science tends to suggest that allowing more than one dimension will have very important consequences. Indeed, it turns out that many of the phenomena that have been most studied in traditional science simply do not occur in just one dimension.

Phenomena that involve geometrical shapes, for example, usually require at least two dimensions, while phenomena that rely on the existence of knotted structures require three dimensions. But what about the phenomenon of complexity? How much does it depend on dimension?

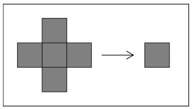
It could be that in going beyond one dimension the character of the behavior that we would see would immediately change. And indeed in the course of this chapter, we will come across many examples of specific effects that depend on having more than one dimension.

But what we will discover in the end is that at an overall level the behavior we see is not fundamentally much different in two or more dimensions than in one dimension. Indeed, despite what we might expect from traditional science, adding more dimensions does not ultimately seem to have much effect on the occurrence of behavior of any significant complexity.

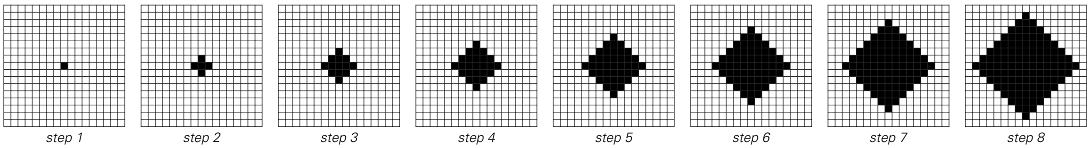
Cellular Automata

The cellular automata that we have discussed so far in this book are all purely one-dimensional, so that at each step, they involve only a single line of cells. But one can also consider two-dimensional cellular automata that involve a whole grid of cells, with the color of each cell being updated according to a rule that depends on its neighbors in all four directions on the grid, as in the picture below.

The form of the rule for a typical two-dimensional cellular automaton. In the cases discussed in this section, each cell is either black or white. Usually I consider so-called totalistic rules in which the new color of the center cell depends only on the average of the previous colors of its four neighbors, as well as on its own previous color.

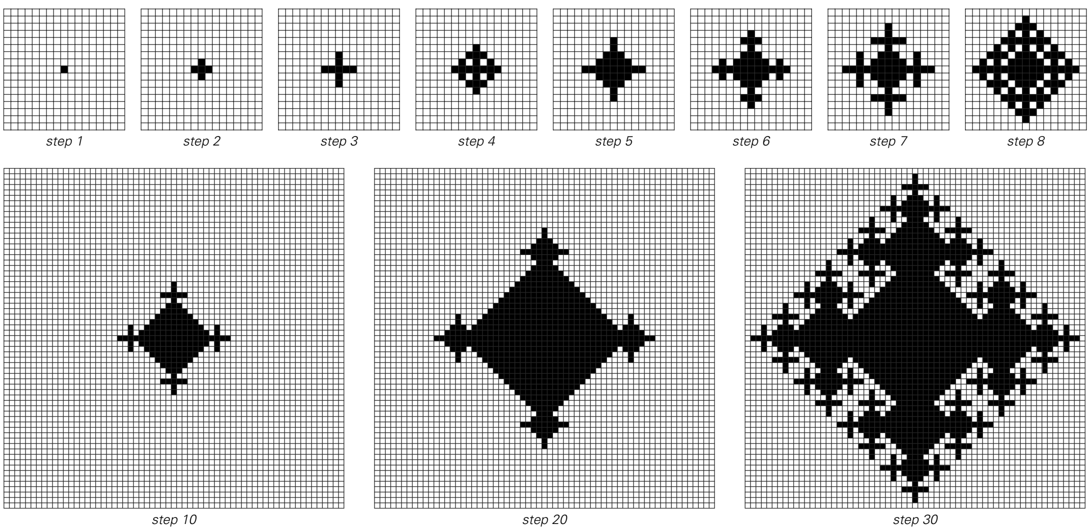


The pictures below show what happens with an especially simple rule in which a particular cell is taken to become black if any of its four neighbors were black on the previous step.



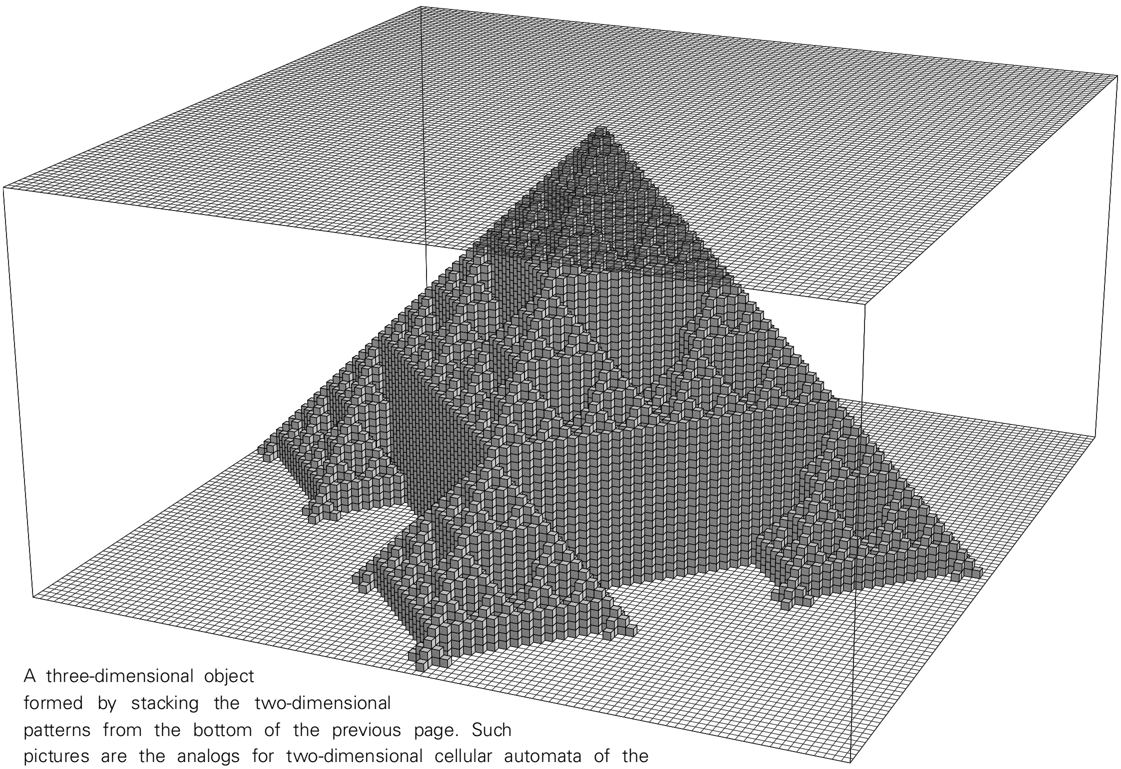
Successive steps in the evolution of a two-dimensional cellular automaton whose rule specifies that a particular cell should become black if any of its neighbors were black on the previous step. (In the numbering scheme described on page 173 this rule is code 1022.)

Starting from a single black cell, this rule just yields a uniformly expanding diamond-shaped region of black cells. But by changing the rule slightly, one can obtain more complicated patterns of growth. The pictures below show what happens, for example, with a rule in which each cell becomes black if just one or all four of its neighbors were black on the previous step, but otherwise stays the same color as it was before.



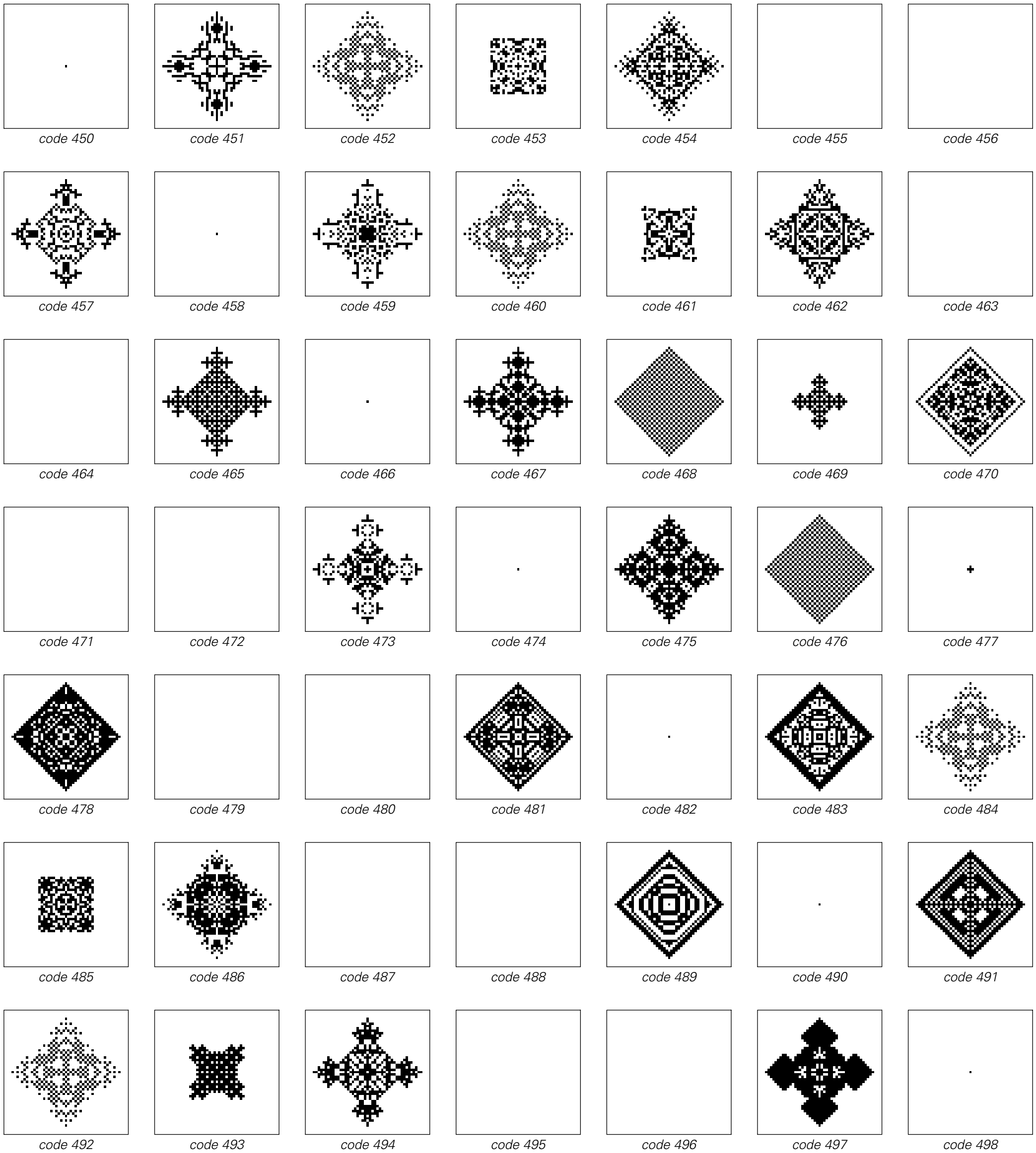
Steps in the evolution of a two-dimensional cellular automaton whose rule specifies that a particular cell should become black if exactly one or all four of its neighbors were black on the previous step, but should otherwise stay the same color. Starting with a single black cell, this rule yields an intricate, if very regular, pattern of growth. (In the numbering scheme on page 173, the rule is code 942.)

The patterns produced in this case no longer have a simple geometrical form, but instead often exhibit an intricate structure somewhat reminiscent of a snowflake. Yet despite this intricacy, the patterns still show great regularity. And indeed, if one takes the patterns from successive steps and stacks them on top of each other to form a three-dimensional object, as in the picture below, then this object has a very regular nested structure.

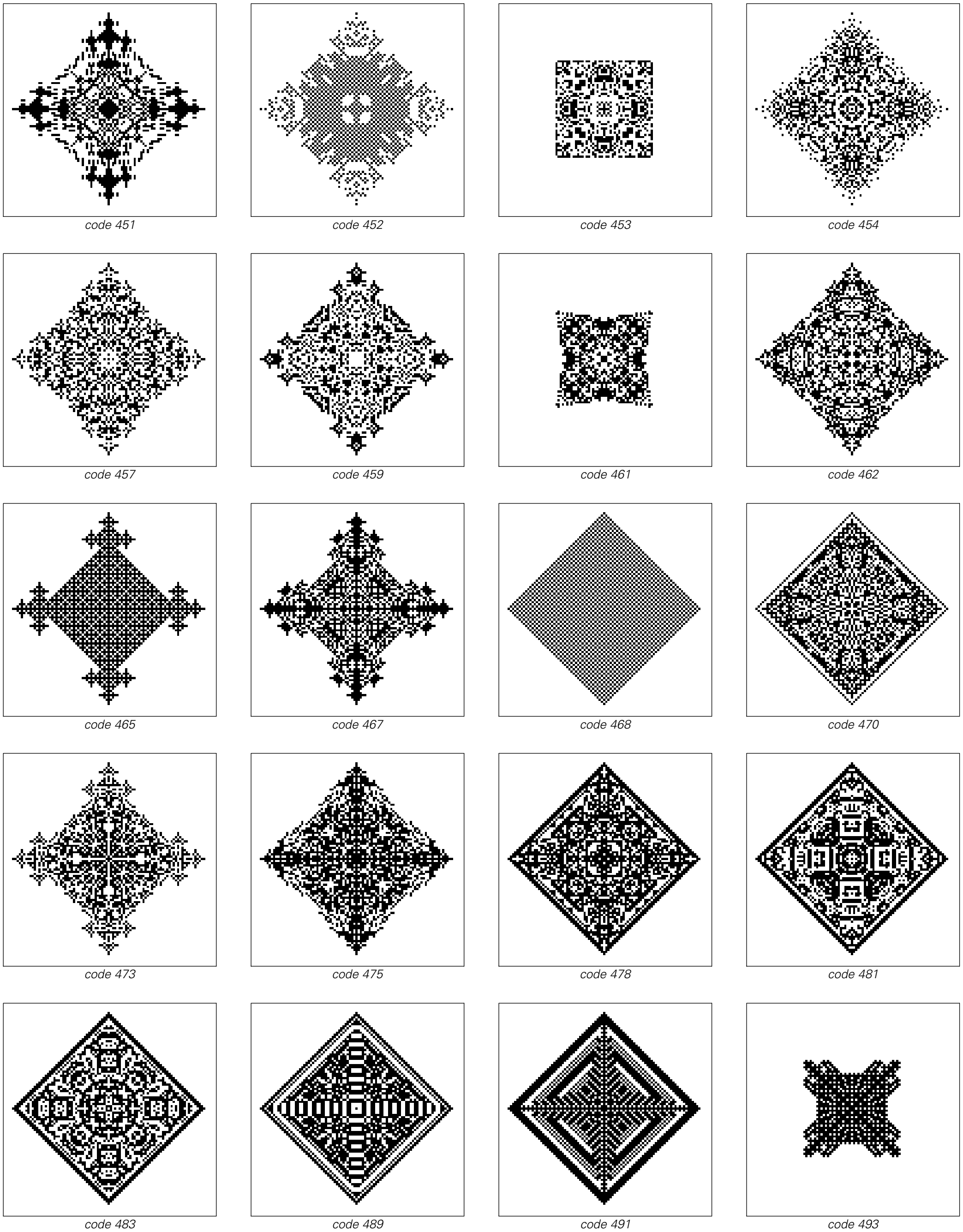


A three-dimensional object formed by stacking the two-dimensional patterns from the bottom of the previous page. Such pictures are the analogs for two-dimensional cellular automata of the two-dimensional pictures that I often generate for one-dimensional cellular automata.

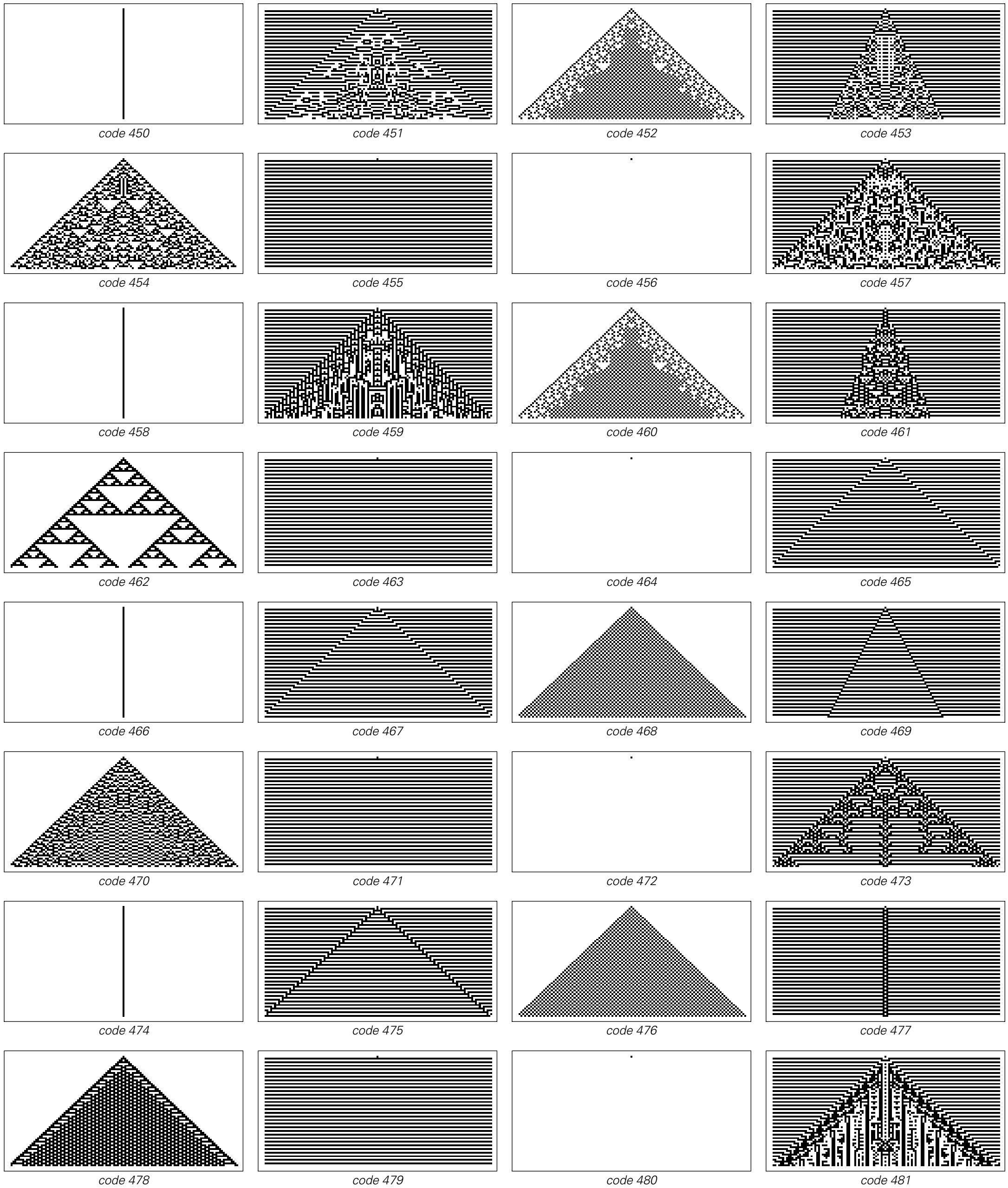
But what about other rules? The facing page and the one that follows show patterns produced by two-dimensional cellular automata with a sequence of different rules. Within each pattern there is often considerable complexity. But this complexity turns out to be very similar to the complexity we have already seen in one-dimensional



Patterns generated by a sequence of two-dimensional cellular automaton rules. The patterns are produced by starting from a single black square and then running for 22 steps. In each case the base 2 digit sequence for the code number specifies the rule as follows. The last digit specifies what color the center cell should be if all its neighbors were white on the previous step, and it too was white. The second-to-last digit specifies what happens if all the neighbors are white, but the center cell itself is black. And each earlier digit then specifies what should happen if progressively more neighbors are black. (Compare page 60.)



Patterns generated by two-dimensional cellular automata from the previous page, but now after twice as many steps.



Evolution of one-dimensional slices through some of the two-dimensional cellular automata from the previous two pages. Each picture shows the colors of cells that lie on the one-dimensional line that goes through the middle of each two-dimensional pattern. The results are strikingly similar to ones we saw in previous chapters in purely one-dimensional cellular automata.

cellular automata. And indeed the previous page shows that if one looks at the evolution of a one-dimensional slice through each two-dimensional pattern the results one gets are strikingly similar to what we have seen in ordinary one-dimensional cellular automata.

But looking at such slices cannot reveal much about the overall shapes of the two-dimensional patterns. And in fact it turns out that for all the two-dimensional cellular automata shown on the last few pages, these shapes are always very regular.

But it is nevertheless possible to find two-dimensional cellular automata that yield less regular shapes. And as a first example, the picture on the facing page shows a rule that produces a pattern whose surface has seemingly random irregularities, at least on a small scale.

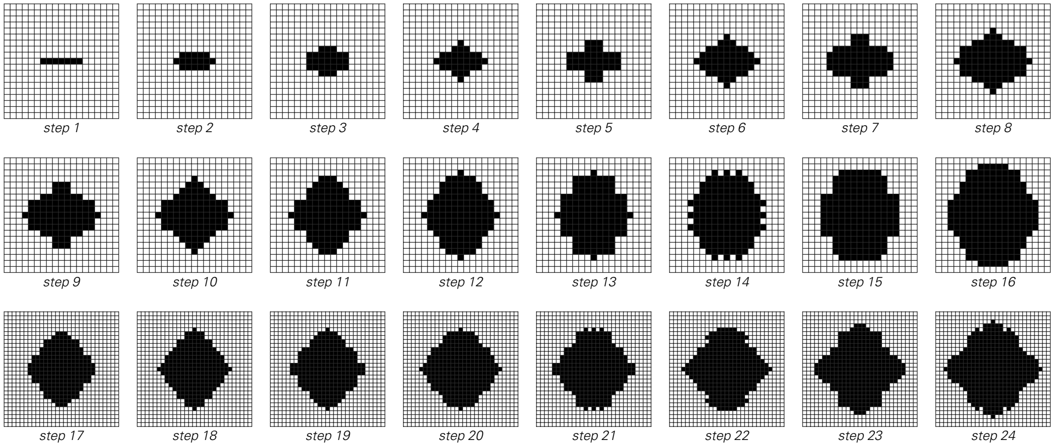
In this particular case, however, it turns out that on a larger scale the surface follows a rather smooth curve. And indeed, as the picture on page 178 shows, it is even possible to find cellular automata that yield overall shapes that closely approximate perfect circles.

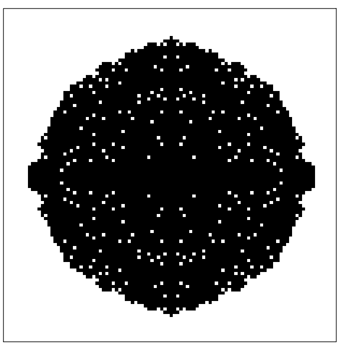
But it is certainly not the case that all two-dimensional cellular automata produce only simple overall shapes. The pictures on pages 179–181 show one rule, for example, that does not. The rule is actually rather simple: it just states that a particular cell should become black whenever exactly three of its eight neighbors—including diagonals—are black, and otherwise it should stay the same color as it was before.

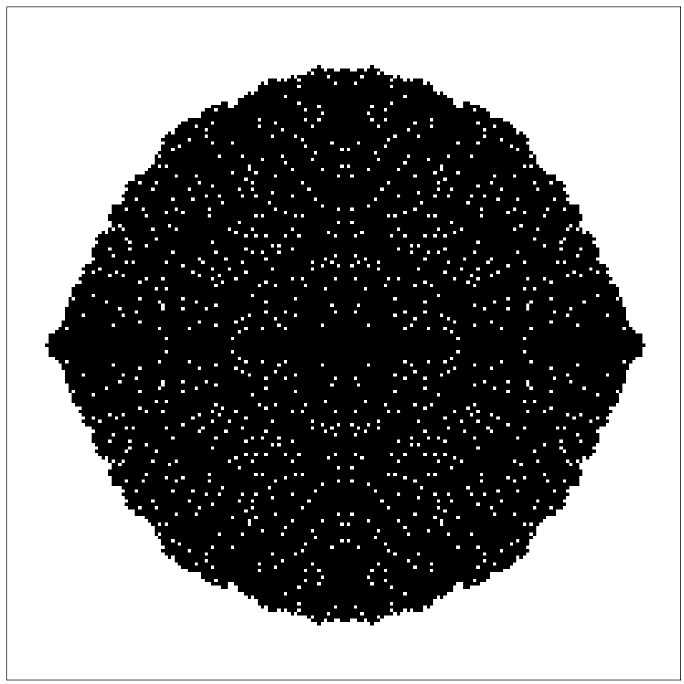
In order to get any kind of growth with this rule one must start with at least three black cells. The picture at the top of page 179 shows what happens with various numbers of black cells. In some cases the patterns produced are fairly simple—and typically stop growing after just a few steps. But in other cases, much more complicated patterns are produced, which often apparently go on growing forever.

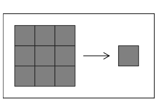
The pictures on page 181 show the behavior produced by starting from a row of eleven black cells, and then evolving for several hundred steps. The shapes obtained seem continually to go on changing, with no simple overall form ever being produced.

And so it seems that there can be great complexity not only in the detailed arrangement of black and white cells in a two-dimensional cellular automaton pattern, but also in the overall shape of the pattern.

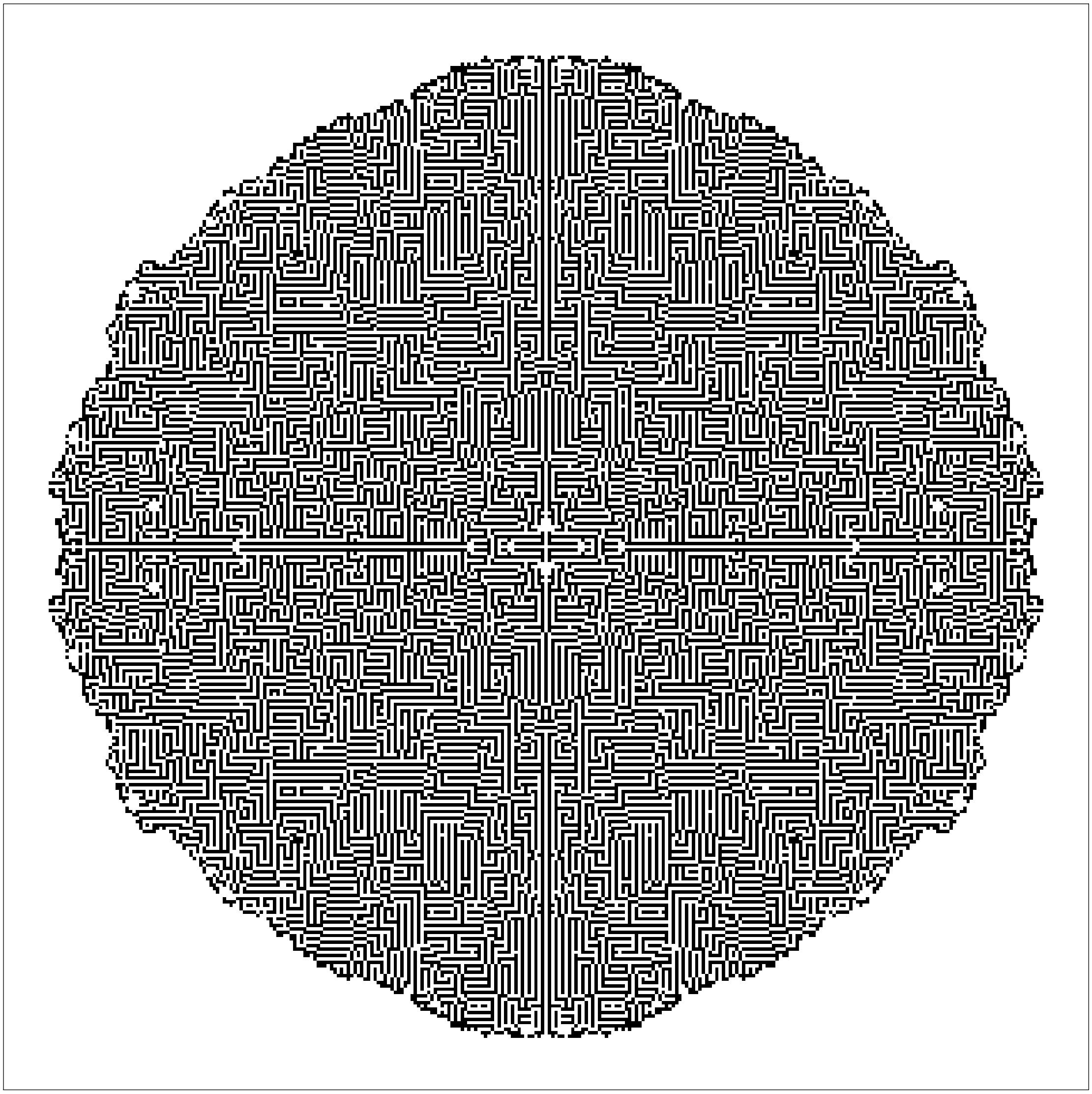



step 100

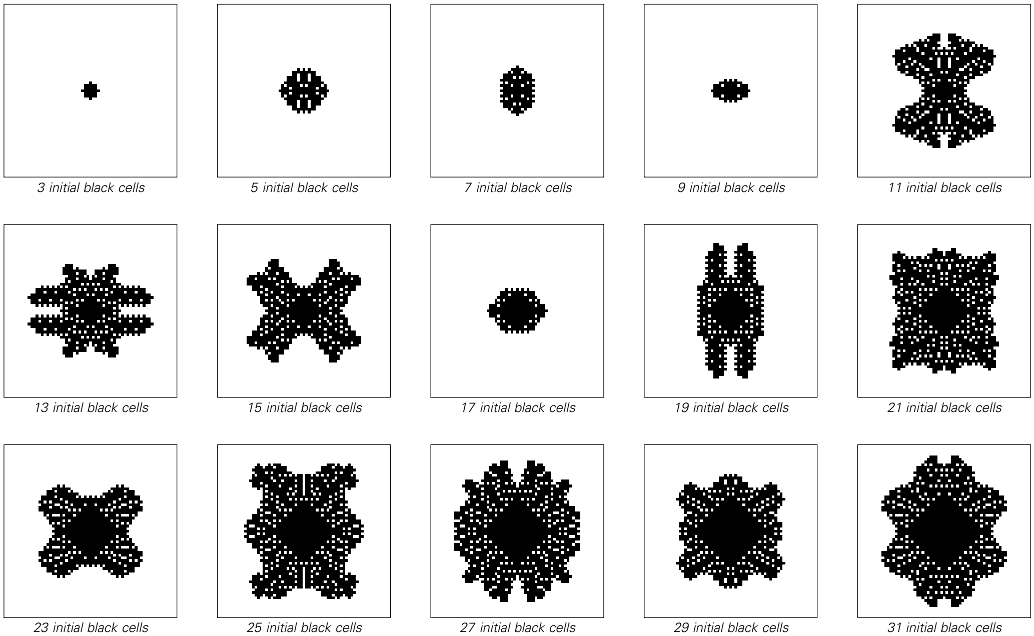

step 200



A two-dimensional cellular automaton that yields a pattern with a rough surface. The rule used here includes diagonal neighbors, and so involves a total of 8 neighbors for each cell, as indicated in the icon on the left. The rule specifies that the center cell should become black if either 3 or 5 of its 8 neighbors were black on the step before, and should otherwise stay the same color as it was before. The initial condition in the case shown consists of a row of 7 black cells. In an extension to 8 neighbors of the scheme used in the pictures a few pages back, the rule has code number 175850.



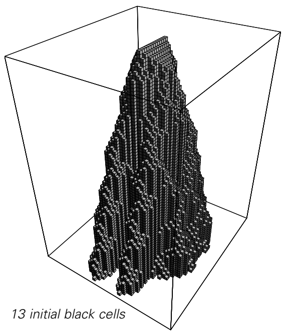
A cellular automaton that yields a pattern whose shape closely approximates a circle. The rule used is of the same kind as on the previous page, but now takes the center cell to become black only if it has exactly 3 black neighbors. If it has 1, 2 or 4 black neighbors then it stays the same color as it was before, and if it has 5 or more black neighbors, then it becomes white on the next step (code number 746). The initial condition consists of a row of 7 black cells, just as in the picture on the previous page. The pattern shown here is the result of 400 steps in the evolution of the system. After t steps, the radius of the approximate circle is about $0.37\,t$.



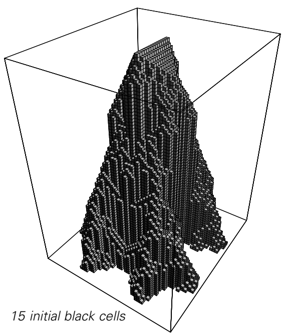
Patterns produced by evolution according to a simple two-dimensional cellular automaton rule starting from rows of black cells of various lengths. The rule used specifies that a particular cell should become black if exactly three out of its eight neighbors (with diagonal neighbors included) are black (code number 174826). The patterns in the picture are obtained by 60 steps of evolution according to this rule. The smaller patterns above have all stopped growing after this number of steps, but many of the other patterns apparently go on growing forever.

So what about three-dimensional cellular automata? It is straightforward to generalize the setup for two-dimensional rules to the three-dimensional case. But particularly on a printed page it is fairly difficult to display the evolution of a three-dimensional cellular automaton in a way that can readily be assimilated.

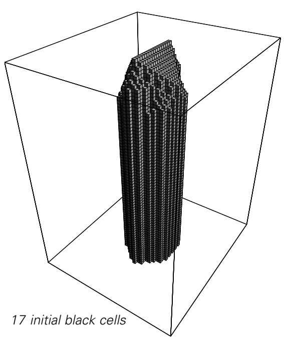
Pages 182 and 183 do however show a few examples of three-dimensional cellular automata. And just as in the two-dimensional case, there are some specific new phenomena that can be seen. But overall it seems that the basic kinds of behavior produced are just the same as in one and two dimensions. And in particular, the basic phenomenon of complexity does not seem to depend in any crucial way on the dimensionality of the system one looks at.



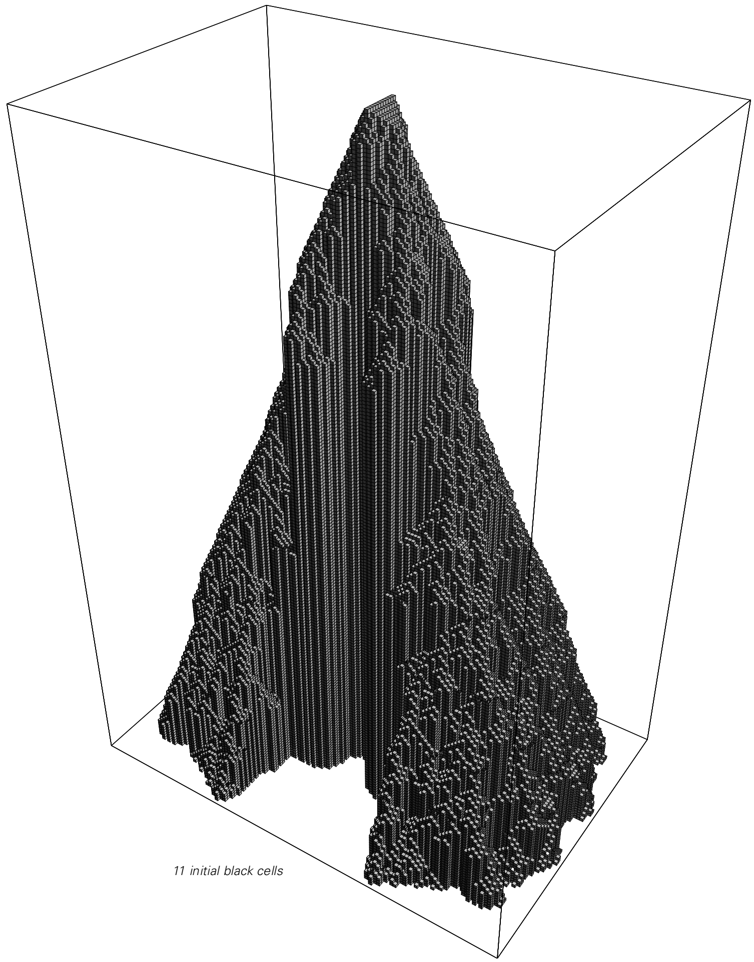
13 initial black cells



15 initial black cells

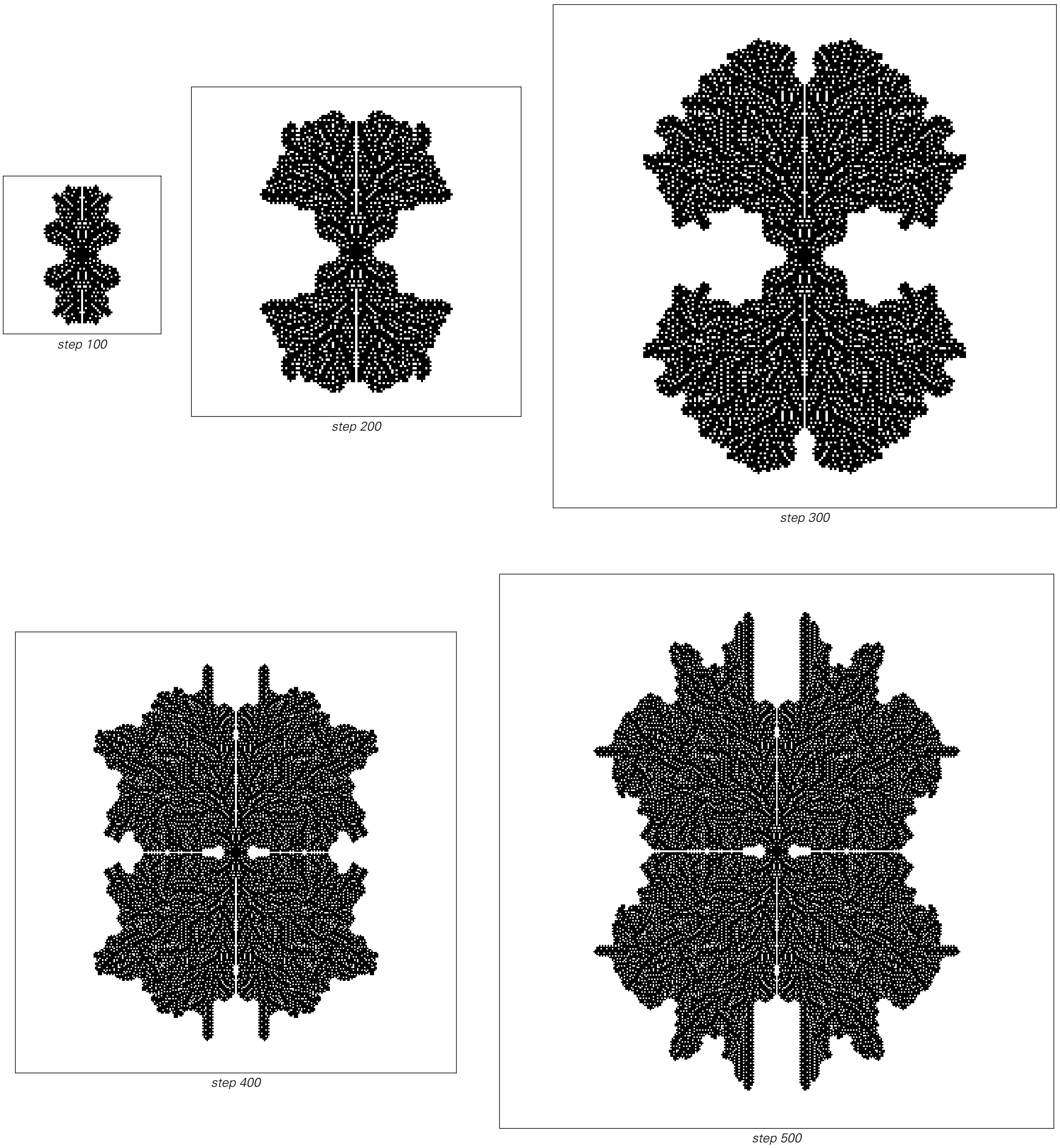


17 initial black cells

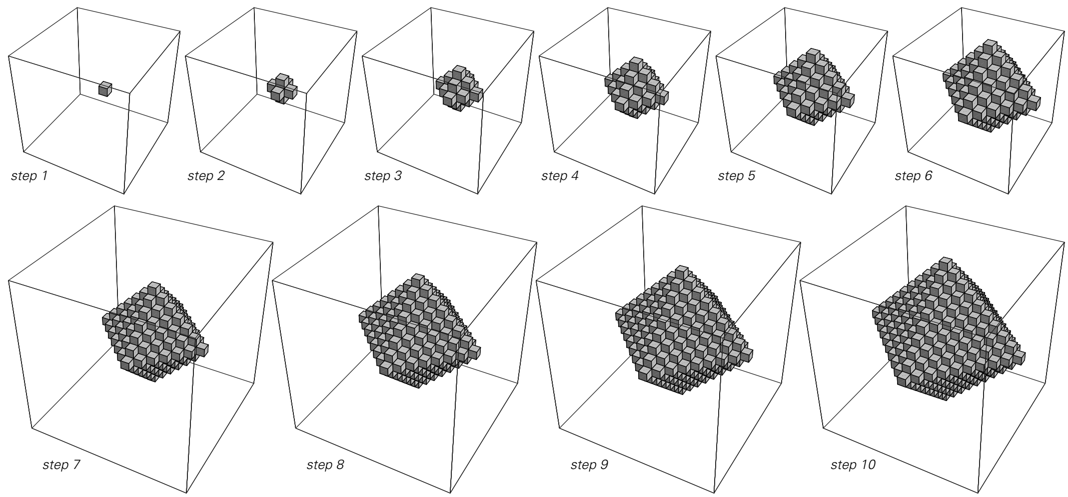


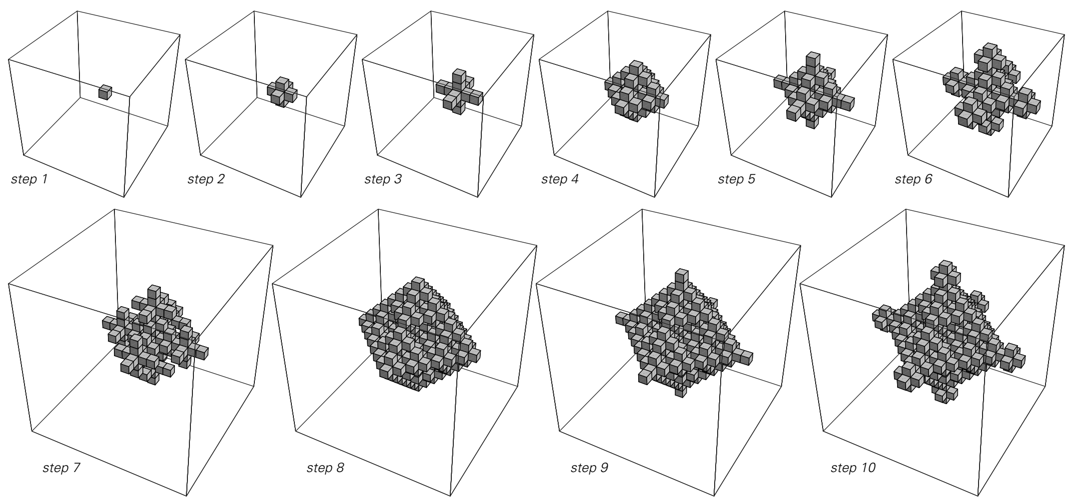
11 initial black cells

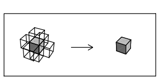
Three-dimensional objects formed by stacking successive two-dimensional patterns produced in the evolution of the cellular automaton from the previous page. The large picture on the right shows 200 steps of evolution.



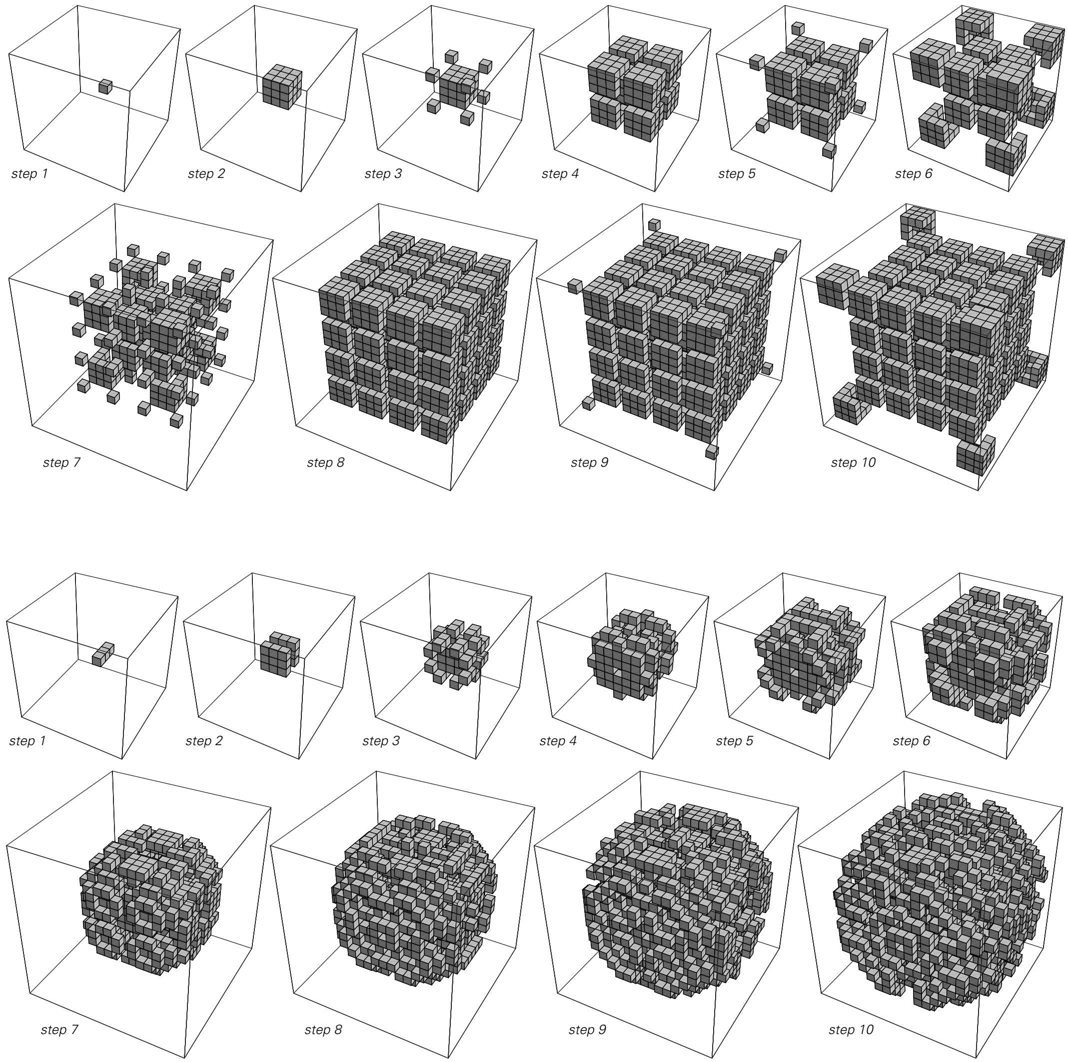
Stages in the evolution of the cellular automaton from the facing page, starting with an initial condition consisting of a row of 11 black cells.

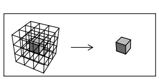






Examples of three-dimensional cellular automata. In the top set of pictures, the rule specifies that a cell should become black whenever any of the six neighbors with which it shares a face were black on the step before. In the bottom pictures, the rule specifies that a cell should become black only when exactly one of its six neighbors was black on the step before. In both cases, the initial condition contains a single black cell. In the top pictures, the limiting shape obtained is a regular octahedron. In the bottom pictures, it is a nested pattern analogous to the two-dimensional one on page 171.





Further examples of three-dimensional cellular automata, but now with rules that depend on all 26 neighbors that share either a face or a corner with a particular cell. In the top pictures, the rule specifies that a cell should become black when exactly one of its 26 neighbors was black on the step before. In the bottom pictures, the rule specifies that a cell should become black only when exactly two of its 26 neighbors were black on the step before. In the top pictures, the initial condition contains a single black cell; in the bottom pictures, it contains a line of three black cells.