EXCERPTED FROM

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A NEW KIND OF SCIENCE

SECTION 8.4

Fluid Flow

Fluid Flow

A great many striking phenomena in nature involve the flow of fluids like air and water—as illustrated on the facing page. Typical of what happens is what one sees when water flows around a solid object. At sufficiently slow speeds, the water in effect just slides smoothly around, yielding a very simple laminar pattern of flow. But at higher speeds, there starts to be a region of slow-moving water behind the object, and a pair of eddies are formed as the water swirls into this region.

As the speed increases, these eddies become progressively more elongated. And then suddenly, when a critical speed is reached, the eddies in effect start breaking off, and getting carried downstream. But every time one eddy breaks off, another starts to form, so that in the end a whole street of eddies are seen in the wake behind the object.

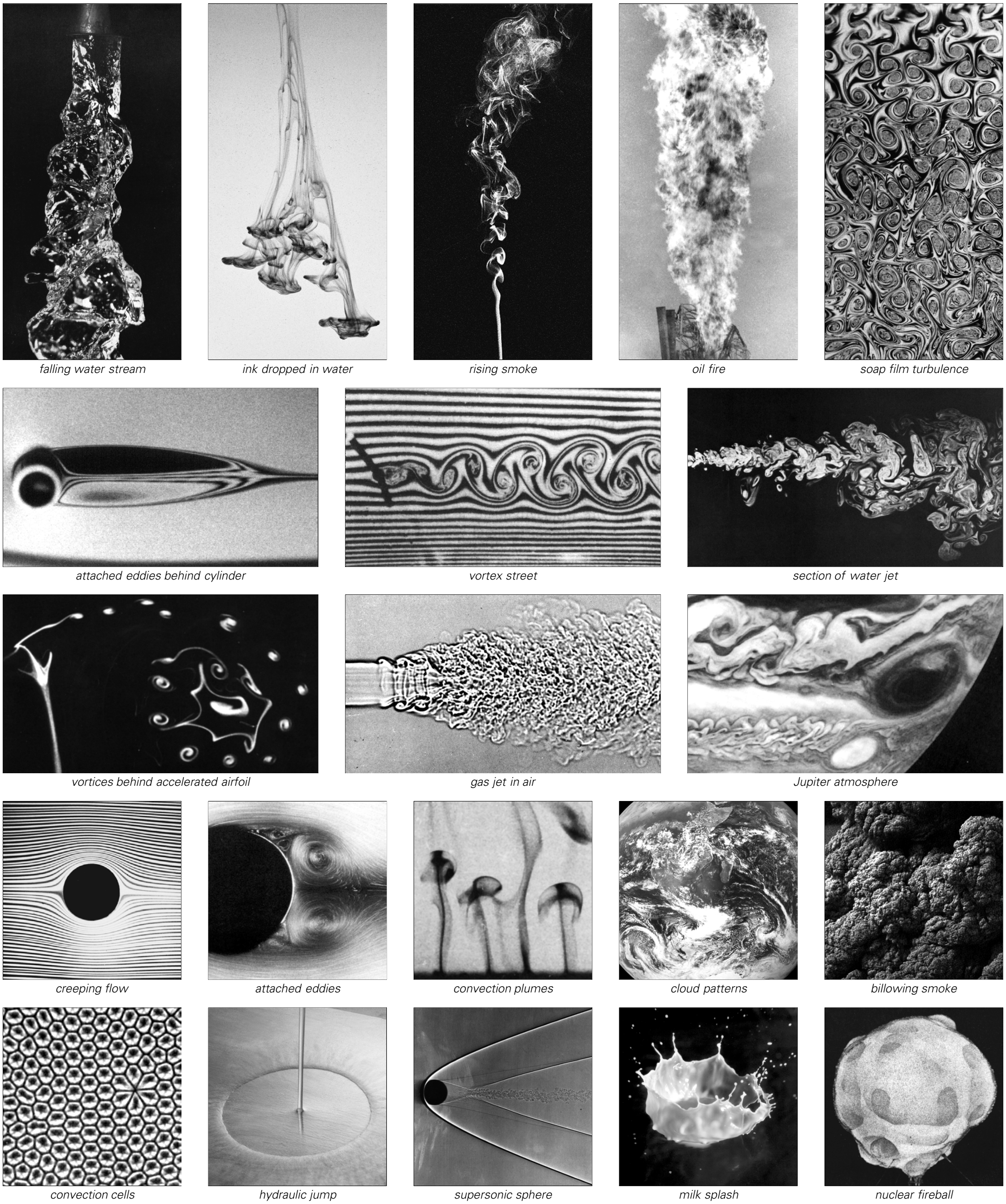
At first, these eddies are arranged in a very regular way. But as the speed of the flow is increased, glitches begin to appear, at first far behind the object, but eventually throughout the wake. Even at the highest speeds, some overall regularity nevertheless remains. But superimposed on this is all sorts of elaborate and seemingly quite random behavior.

But this is just one example of the very widespread phenomenon of fluid turbulence. For as the pictures on the facing page indicate—and as common experience suggests—almost any time a fluid is made to flow rapidly, it tends to form complex patterns that seem in many ways random.

So why fundamentally does this happen?

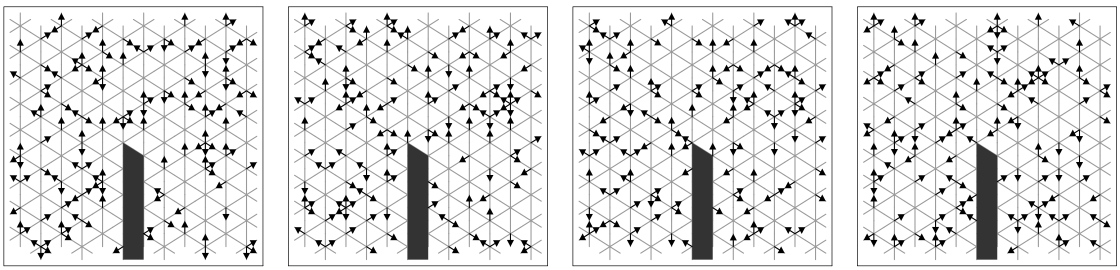
Traditional science, with its basis in mathematical equations, has never really been able to provide any convincing underlying explanation. But from my discovery that complex and seemingly random behavior is in a sense easy to get even with very simple programs, the phenomenon of fluid turbulence immediately begins to seem much less surprising.

But can simple programs really reproduce the particular kinds of behavior we see in fluids? At a microscopic level, physical fluids consist of large numbers of molecules moving around and colliding with each other. So as a simple idealization, one can consider having a large number of particles move around on a fixed discrete grid, and undergo collisions governed by simple cellular-automaton-like rules.

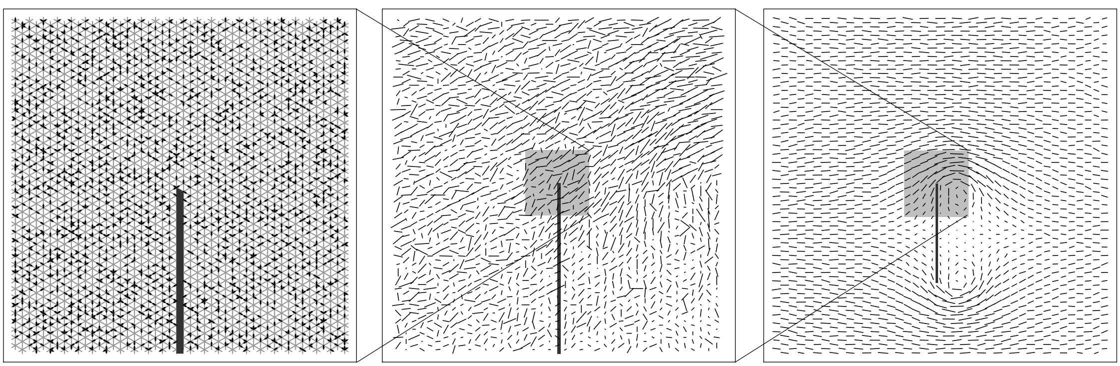


Examples of typical patterns generated in various kinds of fluid flow. Note the frequent occurrence of seemingly random turbulence.

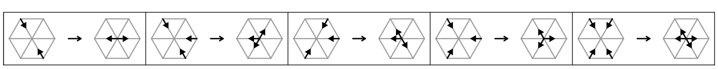
The pictures below give an example of such a system. In the top row of pictures—as well as picture (a)—all one sees is a collection of discrete particles bouncing around. But if one zooms out, and looks at average motion of increasingly large blocks of particles—as in pictures (b) and (c)—then what begins to emerge is behavior that seems smooth and continuous—just like one expects to see in a fluid.



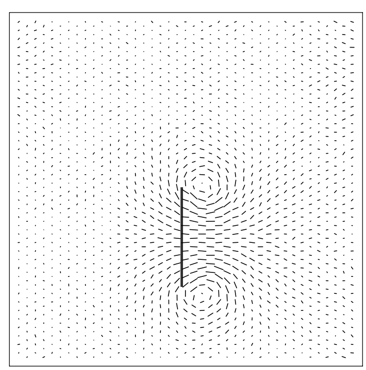
step 1000 *step 1001* *step 1002* *step 1003*



(a) individual cells *(b) 5×5 averages* *(c) 25×25 averages*



A simple cellular automaton system set up to emulate the microscopic behavior of molecules in a fluid. At each step the configuration of particles is updated according to the simple collision rules shown above. Particles are reflected whenever they hit the plate. A steady stream of particles is inserted in a regular way far to the left, with an average speed 3/10 of the maximum possible. Picture (a) shows the configuration of individual particles; pictures (b) and (c) show total velocities of successively larger blocks of particles. Picture (d) is obtained by transforming to a reference frame in which the fluid is on average at rest.



(d) moving with fluid

This happens for exactly the same reason as in a real fluid, or, for that matter, in various examples that we saw in Chapter 7: even though at an underlying level the system consists of discrete particles, the effective randomness of the detailed microscopic motions of these particles makes their large-scale average behavior seem smooth and continuous.

We know from physical experiments that the characteristics of fluid flow are almost exactly the same for air, water, and all other ordinary fluids. Yet at an underlying level these different fluids consist of very different kinds of molecules, with very different properties. But somehow the details of such microscopic structure gets washed out if one looks at large-scale fluid-like behavior.

Many times in this book we have seen examples where different systems can yield very much the same overall behavior, even though the details of their underlying rules are quite different. But in the particular case of systems like fluids, it turns out that one can show—as I will discuss in the next chapter—that so long as certain physical quantities such as particle number and momentum are conserved, then whenever there is sufficient microscopic randomness, it is almost inevitable that the same overall fluid behavior will be obtained.

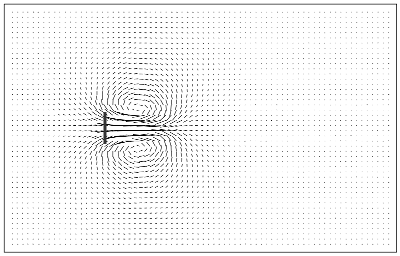
So what this means is that to reproduce the observed properties of physical fluids one should not need to make a model that involves realistic molecules: even the highly idealized particles on the facing page should give rise to essentially the same overall fluid behavior.

And indeed in pictures (c) and (d) one can already see the formation of a pair of eddies, just as in one of the pictures on page 377.

So what happens if one increases the speed of the flow? Does one see the same kinds of phenomena as on page 377? The pictures on the next page suggest that indeed one does. Below a certain critical speed, a completely regular array of eddies is formed. But at the speed used in the pictures on the next page, the array of eddies has begun to show random irregularities just like those associated with turbulence in real fluids.

So where does this randomness come from?

In the past couple of decades it has come to be widely believed that randomness in turbulent fluids must somehow be associated with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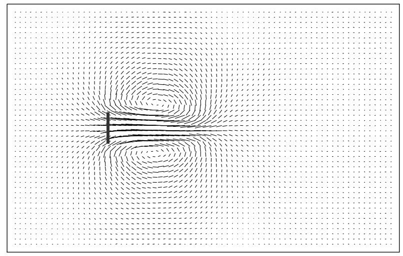
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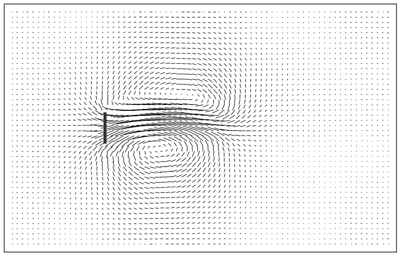
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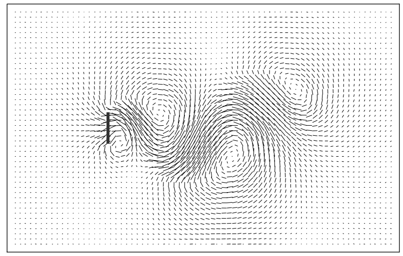
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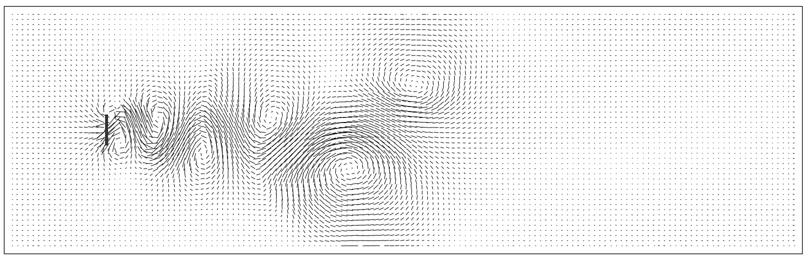
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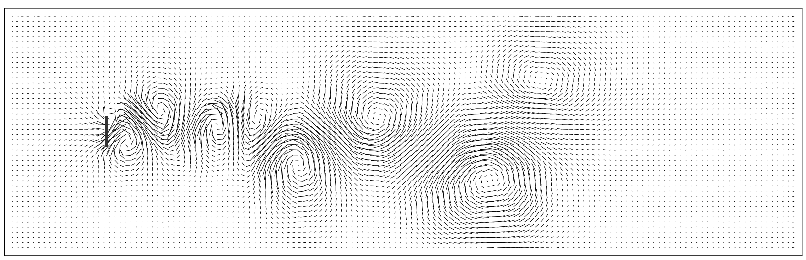
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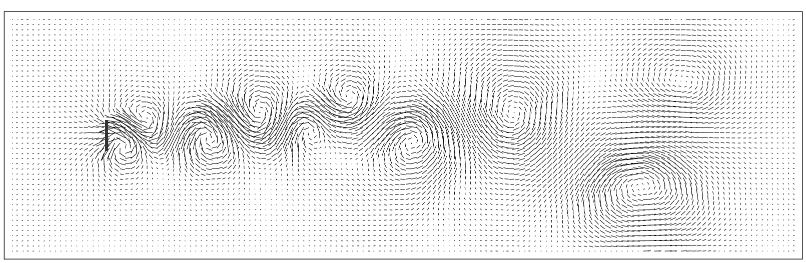

step 70000

A larger example of the cellular automaton system shown on the previous page. In each picture there are a total of 30 million underlying cells. The individual velocity vectors drawn correspond to averages over 20 × 20 blocks of cells. Particles are inserted in a regular way at the left-hand end so as to maintain an overall flow speed equal to about 0.4 of the maximum possible. To make the patterns of flow easier to see, the velocities shown are transformed so that the fluid is on average at rest, and the plate is moving. The underlying density of particles is approximately 1 per cell, or 1/6 the maximum possible—a density which more or less minimizes the viscosity of the fluid. The Reynolds number of the flow shown is then approximately 100. The agreement with experimental results on actual fluid flows is striking.

sensitive dependence on initial conditions, and with the chaos phenomenon that we discussed in Chapter 4.

But while there are certainly mathematical equations that exhibit this phenomenon, none of those typically investigated have any close connection to realistic descriptions of fluid flow.

And in the model on the facing page it turns out that there is essentially no sensitive dependence on initial conditions, at least at the level of overall fluid behavior. If one looks at individual particles, then changing the position of even one particle will typically have an effect that spreads rapidly. But if one looks instead at the average behavior of many particles, such effects get completely washed out. And indeed when it comes to large-scale fluid behavior, it seems to be true that in almost all cases there is no discernible difference between what happens with different detailed initial conditions.

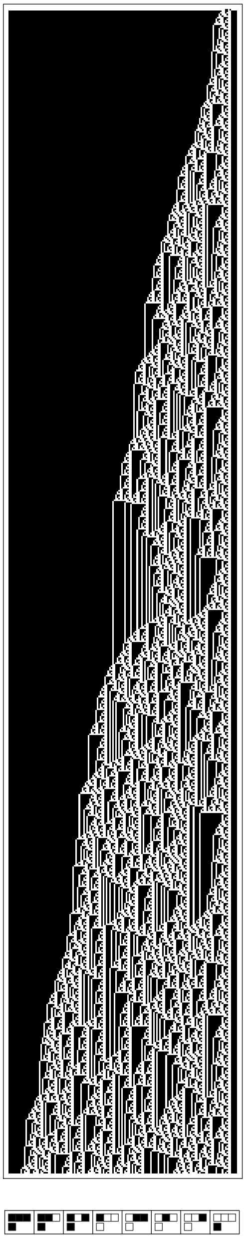
So is there ever sensitive dependence on initial conditions?

Presumably there do exist situations in which there is some kind of delicate balance—say of whether the first eddy is shed at the top or bottom of an object—and in which small changes in initial conditions can have a substantial effect. But such situations appear to be very much the exception rather than the rule. And in the vast majority of cases, small changes instead seem to damp out rapidly—just as one might expect from everyday experience with viscosity in fluids.

So what this means is that the randomness we observe in fluid flow cannot simply be a reflection of randomness that is inserted through the details of initial conditions. And as it turns out, in the pictures on the facing page, the initial conditions were specifically set up to be very simple. Yet despite this, there is still apparent randomness in the overall behavior that is seen.

And so, once again, just as for many other systems that we have studied in this book, there is little choice but to conclude that in a turbulent fluid most of the randomness we see is not in any way inserted from outside but is instead intrinsically generated inside the system itself. In the pictures on page 378 considerable randomness was already evident at the level of individual particles. But since changes in the configurations of such particles do not seem to have any discernible effect on overall patterns of flow, one cannot realistically attribute the large-scale randomness that one sees in a turbulent fluid to randomness that exists at the level of individual particles.

Instead, what seems to be happening is that intrinsic randomness generation occurs directly at the level of large-scale fluid motion. And as an example of a simple approach to modelling this, one can consider having a collection of discrete eddies that occur at discrete positions in the fluid, and interact through simple cellular automaton rules.



A cellular automaton (rule 225) whose behavior is reminiscent of turbulent fluid flow.

The picture on the left shows an example of what can happen. And although many details are different from what one sees in real fluids, the overall mixture of regularity and randomness is strikingly similar.

One consequence of the idea that there is intrinsic randomness generation in fluids and that it occurs at the level of large-scale fluid motion is that with sufficiently careful preparation it should be possible to produce patterns of flow that seem quite random but that are nevertheless effectively repeatable—so that they look essentially the same on every successive run of an experiment.

And even if one looks at existing experiments on fluid flow, there turn out to be quite a few instances—particularly for example involving interactions between small numbers of vortices—where there are known patterns of fluid flow that look intricate, but are nevertheless essentially repeatable. And while none of these yet look complicated enough that they might reasonably be called random, I suspect that in time similar but vastly more complex examples will be found.

Among the patterns of fluid flow on page 377 each has its own particular details and characteristics. But while some of the simpler ones have been captured quite completely by methods based on traditional mathematical equations, the more complex ones have not. And in fact from the perspective of this book this is not surprising.

But now from the experience and intuition developed from the discoveries in this book, I expect that there will in fact be remarkably simple programs that can be found that will successfully manage to reproduce the main features of even the most intricate and apparently random forms of fluid flow.