EXCERPTED FROM

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A NEW KIND OF SCIENCE

SECTION 9.2

The Notion of Reversibility

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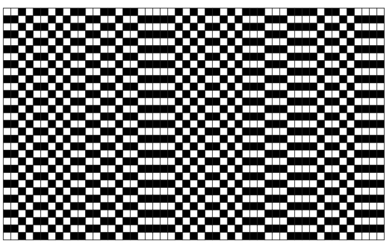
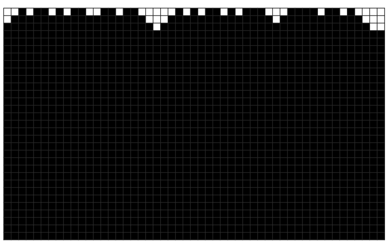
At any particular step in the evolution of a system like a cellular automaton the underlying rule for the system tells one how to proceed to the next step. But what if one wants to go backwards? Can one deduce from the arrangement of black and white cells at a particular step what the arrangement of cells must have been on previous steps?

All current evidence suggests that the underlying laws of physics have this kind of reversibility. So this means that given a sufficiently precise knowledge of the state of a physical system at the present time, it is therefore possible to deduce not only what the system will do in the future, but also what it did in the past.

In the first cellular automaton shown below it is also straightforward to do this. For any cell that has one color at a particular step must always have had the opposite color on the step before.



rule 51



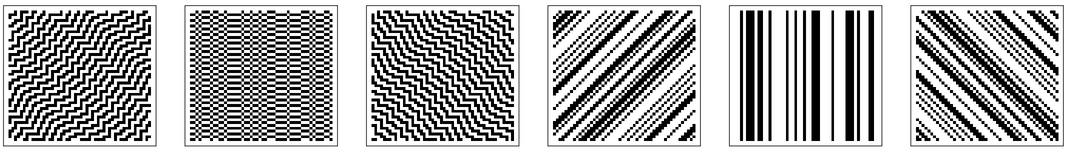
rule 254

Examples of cellular automata that are and are not reversible. Rule 51 is reversible, so that it preserves enough information to allow one to go backwards from any particular step as well as forwards. Rule 254 is not reversible, since it always evolves to uniform black and preserves no information about the arrangement of cells on earlier steps.

But the second cellular automaton works differently, and does not allow one to go backwards. For after just a few steps, it makes every cell black, regardless of what it was before—with the result that there is no way to tell what color might have occurred on previous steps.

There are many examples of systems in nature which seem to organize themselves a little like the second case above. And indeed the conflict between this and the known reversibility of underlying laws of physics is related to the subject of the next section in this chapter.

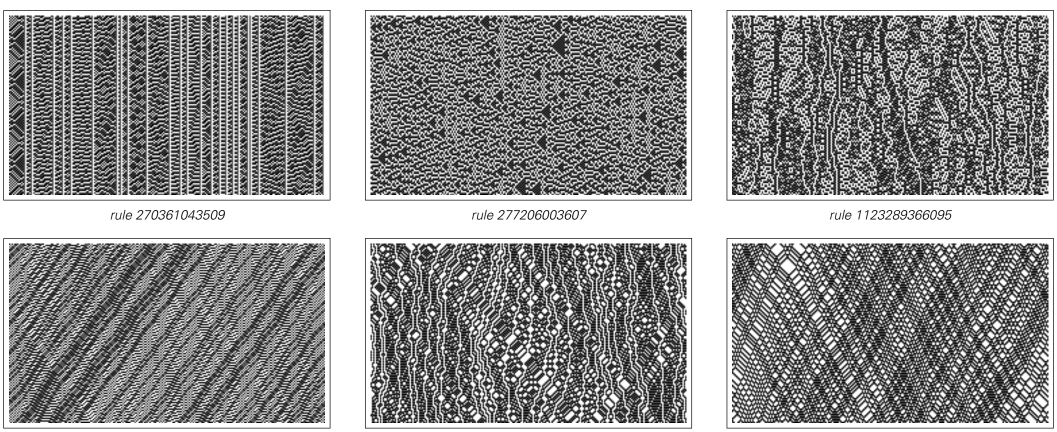
But my purpose here is to explore what kinds of systems can be reversible. And of the 256 elementary cellular automata with two colors and nearest-neighbor rules, only the six shown below turn out to be reversible. And as the pictures demonstrate, all of these exhibit fairly trivial behavior, in which only rather simple transformations are ever made to the initial configuration of cells.



rule 15 *rule 51* *rule 85* *rule 170* *rule 204* *rule 240*

Examples of the behavior of the six elementary cellular automata that are reversible. In all cases the transformations made to the initial conditions are simple enough that it is straightforward to go backwards as well as forwards in the evolution.

So is it possible to get more complex behavior while maintaining reversibility? There are a total of 7,625,597,484,987 cellular automata with three colors and nearest-neighbor rules, and searching through these one finds just 1800 that are reversible. Of these 1800, many again exhibit simple behavior, much like the pictures above. But some exhibit more complex behavior, as in the pictures below.



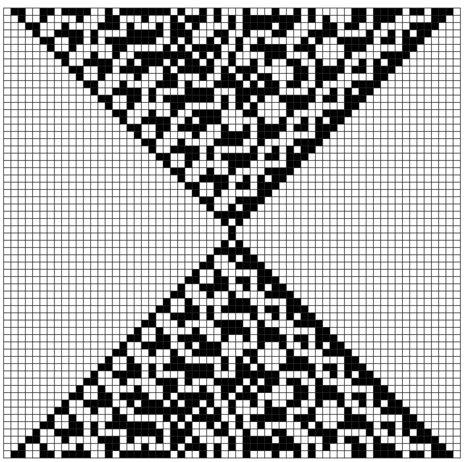
rule 270361043509 *rule 277206003607* *rule 1123289366095*

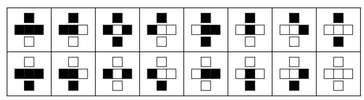
rule 1123956776897 *rule 3097483878567* *rule 3681848058291*

Examples of some of the 1800 reversible cellular automata with three colors and nearest-neighbor rules. Even though these systems exhibit complex behavior that scrambles the initial conditions, all of them are still reversible, so that starting from the configuration of cells at the bottom of each picture, it is always possible to deduce the configurations on all previous steps.

How can one now tell that such systems are reversible? It is no longer true that their evolution leads only to simple transformations of the initial conditions. But one can still check that starting with the specific configuration of cells at the bottom of each picture, one can evolve backwards to get to the top of the picture. And given a particular rule it turns out to be fairly straightforward to do a detailed analysis that allows one to prove or disprove its reversibility.

But in trying to understand the range of behavior that can occur in reversible systems it is often convenient to consider classes of cellular automata with rules that are specifically constructed to be reversible. One such class is illustrated below. The idea is to have rules that explicitly remain the same even if they are turned upside-down, thereby interchanging the roles of past and future.



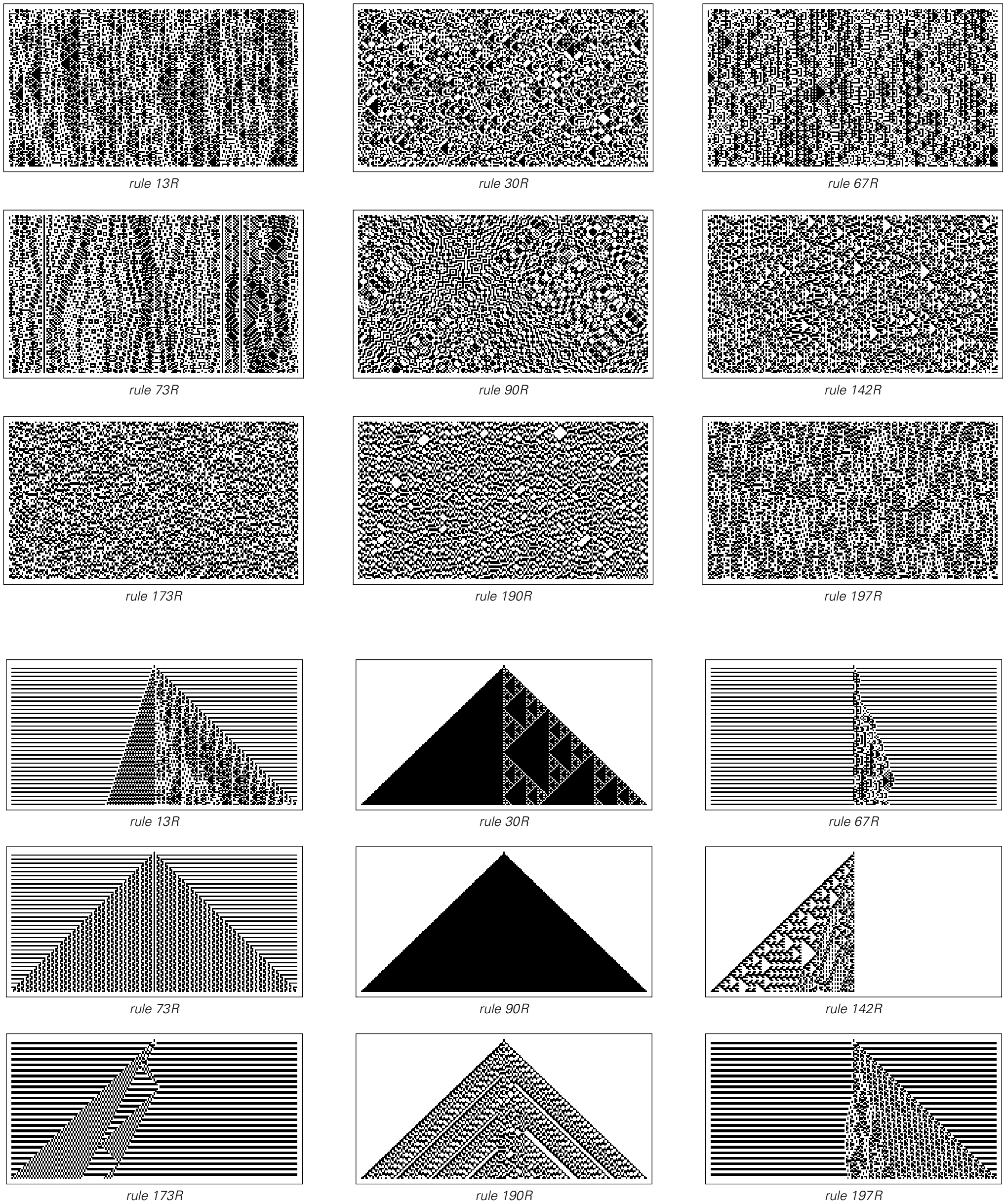


An example of a cellular automaton that is explicitly set up to be reversible. The rule for the system remains unchanged if all its elements are turned upside-down—effectively interchanging the roles of past and future. Patterns produced by the rule must exhibit the same time reversal symmetry, as shown on the left. The specific rule used here is based on taking elementary rule 214, then adding the specification that the new color of a cell should be inverted whenever the cell was black two steps back. Note that by allowing a total of four rather than two colors, a version of the rule that depends only on the immediately preceding step can be constructed.

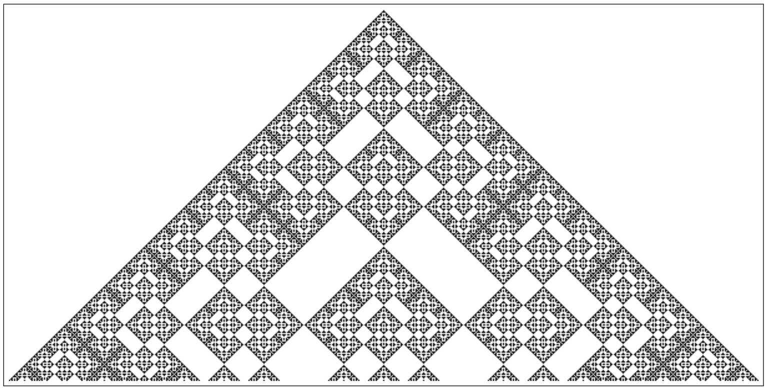
Such rules can be constructed by taking ordinary cellular automata and adding dependence on colors two steps back.

The resulting rules can be run both forwards and backwards. In each case they require knowledge of the colors of cells on not one but two successive steps. Given this knowledge, however, the rules can be used to determine the configuration of cells on either future or past steps.

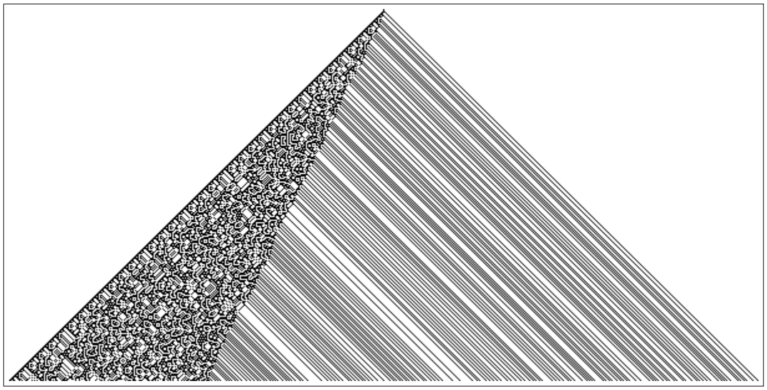
The next two pages show examples of the behavior of such cellular automata with both random and simple initial conditions.



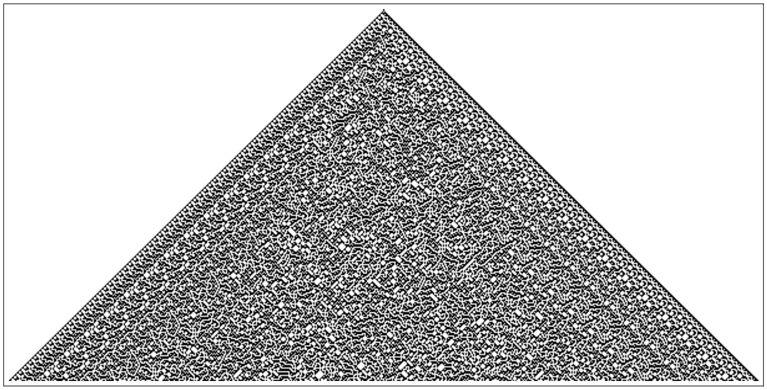
Examples of reversible cellular automata starting from random and from simple initial conditions. In the upper block of pictures, every cell is chosen to be black or white with equal probability on the two successive first steps. In the lower block of pictures, only the center cell is taken to be black on these steps.



rule 150R

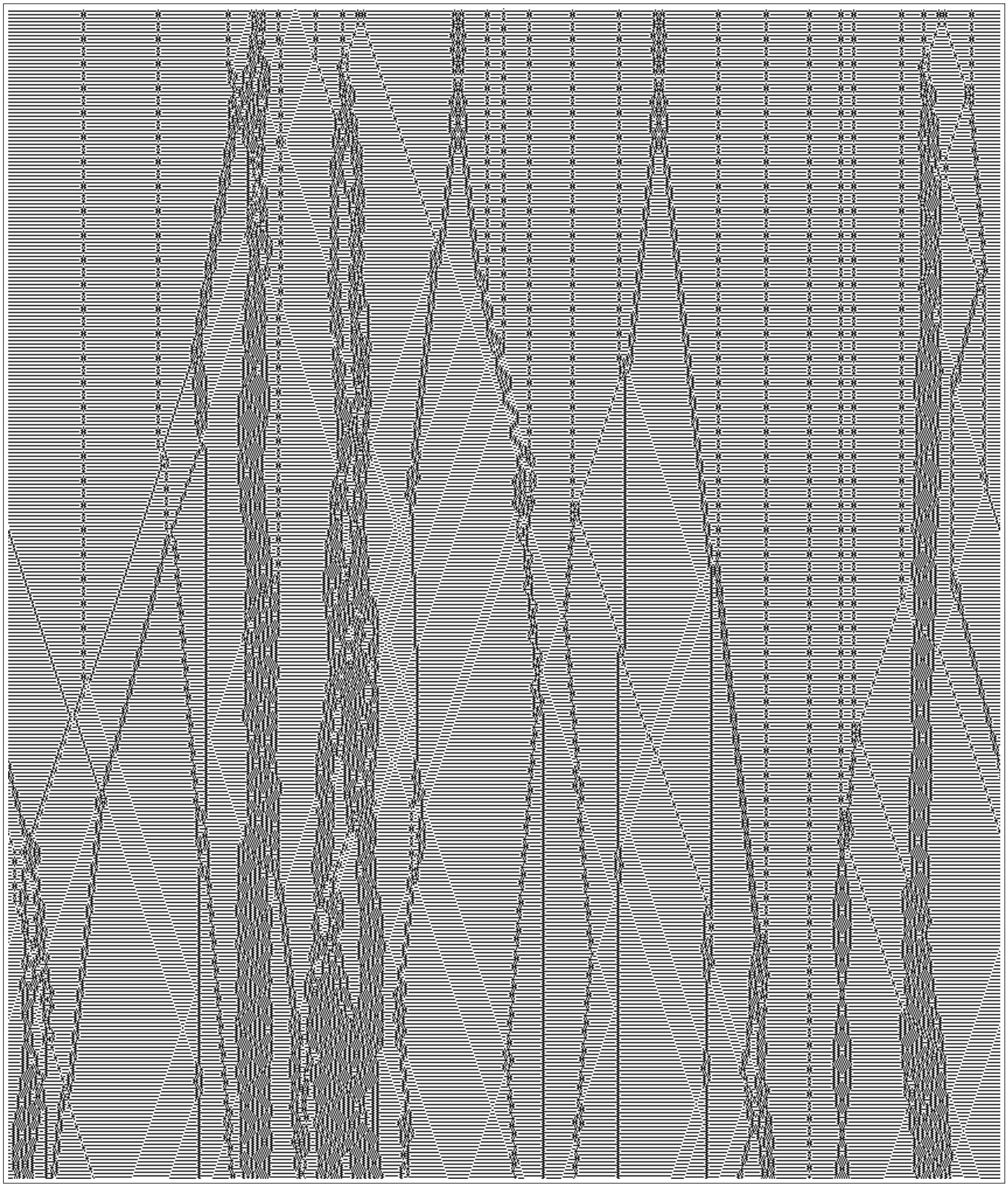


rule 154R



rule 214R

The evolution of three reversible cellular automata for 300 steps. In the first case, a regular nested pattern is obtained. In the other cases, the patterns show many features of randomness.



rule 37R

An example of a reversible cellular automaton whose evolution supports localized structures. Because of the reversibility of the underlying rule, every collision must be able to occur equally well when its initial and final states are interchanged.

In some cases, the behavior is fairly simple, and the patterns obtained have simple repetitive or nested structures. But in many cases, even with simple initial conditions, the patterns produced are highly complex, and seem in many respects random.

The reversibility of the underlying rules has some obvious consequences, such as the presence of triangles pointing sideways but not down. But despite their reversibility, the rules still manage to produce the kinds of complex behavior that we have seen in cellular automata and many other systems throughout this book.

So what about localized structures?

The picture on the facing page demonstrates that these can also occur in reversible systems. There are some constraints on the details of the kinds of collisions that are possible, but reversible rules typically tend to work very much like ordinary ones.

So in the end it seems that even though only a very small fraction of possible systems have the property of being reversible, such systems can still exhibit behavior just as complex as one sees anywhere else.

Irreversibility and the Second Law of Thermodynamics

All the evidence we have from particle physics and elsewhere suggests that at a fundamental level the laws of physics are precisely reversible. Yet our everyday experience is full of examples of seemingly irreversible phenomena. Most often, what happens is that a system which starts in a fairly regular or organized state becomes progressively more and more random and disorganized. And it turns out that this phenomenon can already be seen in many simple programs.

The picture at the top of the next page shows an example based on a reversible cellular automaton of the type discussed in the previous section. The black cells in this system act a little like particles which bounce around inside a box and interact with each other when they collide.

At the beginning the particles are placed in a simple arrangement at the center of the box. But over the course of time the picture shows that the arrangement of particles becomes progressively more random.